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Asymptotic approximation method of force reconstruction: Proof of concept

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ABSTRACT

An important problem in engineering is the determination of the system input based on the system response. This type of problem is difficult to solve as it is often ill-defined, and produces inaccurate or non-unique results. Current reconstruction techniques typically involve the employment of optimization methods or additional constraints to regularize the problem, but these methods are not without their flaws as they may be sub-optimally applied and produce inadequate results. An alternative approach is developed that draws upon concepts from control systems theory, the equilibrium analysis of linear dynamical systems with time-dependent inputs, and asymptotic approximation analysis. This paper presents the theoretical development of the proposed method. A simple application of the method is presented to demonstrate the procedure. A more complex application to a continuous system is performed to demonstrate the applicability of the method.

1. Introduction

In many engineering applications, knowledge of the input or applied force is critical to the analysis of a system or structure. Unfortunately, at times, this knowledge may be difficult to obtain as it may not be possible to measure the applied force directly, and we must rely on measurements of the system response to determine the input or force applied to a system. This presents the problem of developing a model that estimates the input based on the measured output. Various approaches to this problem are outlined in [19], where such reconstruction methods are divided into three categories: direct methods, regularization methods, and probabilistic/statistical methods.

Direct methods involve the application of the governing equation explicitly to determine an estimate for the applied force. These methods were employed by Jacquelin and Hamelin [11] to a split Hopkinson pressure bar using strain measurements, Law et al. [10] to an Euler-Bernoulli beam with a moving force applied, and were more generally developed by Karlsson [14], and Avitabile et al. [1]. While these methods are simplest to apply, they generally are ill-conditioned and can produce inadequate results.

To overcome this ill-conditioning, additional constraints are often applied to transform the system into a well-conditioned problem, which is known as regularization. Various methods have been developed to regularize a problem, for example, through modal truncation, and optimization. Methods of truncation involve the isolation and removal of characteristics deemed to be attributed to noise, such as mode selection techniques developed by Jiang and Hu [12,13], and the

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application of wavenumber filtering by Djamaa et al. [4]. Additionally, the application of optimization methods were presented by Huang et al. [9] that utilized and optimized additional terms to filter noise inherent in the system, and the conjugate gradient method was detailed by Huang [8] and Yen and Wu [22,23]. Another type of optimization solution involves the application of the Levenberg-Marquardt iterative regularization method to solve inverse problems, and were presented by Gunawan [7] and Gasparo et al. [5]. While the application of these methods are generally successful, they do have their drawbacks. Some regularization methods possess additional parameters for regularization that lack a clearly defined method of determination while others require an intense level of computation to produce an estimate of the applied force.

In addition to regularization methods, probabilistic/statistical methods have been applied to the solution of the inverse problem. These methods generally incorporate uncertainties such as measurement or system noise into the analysis and allows for a more holistic perspective of the inverse problem. One method that is becoming prevalent is the application of the Kalman filter to the inverse problem, and has been applied in conjunction with a recursive least squares algorithm by Ma et al. [16], by Xu et al. [21], and in an augmented form by Lourens et al. [15]. While the application of these methods have proven successful, like regularization methods, probabilistic methods can be computationally intensive as they may require a significant amount of data and time for computation.

Fundamentally, these methods of estimation involve the truncation of system characteristics attributed to noise or the development of an optimization problem to produce a well-defined formulation. The dismissal of these assumptions would allow more freedom in the estimation, and allow us to explore new avenues of analysis that have otherwise gone unexplored. Our objective in developing our dynamic force estimation method was to develop a procedure that is inherently stable independent of any optimization or assumptions on the characteristics of the temporal component of the applied force throughout the recovery process.

The work presented in this paper should be seen as a proof-of-concept where the estimation of dynamic forces is viewed from an asymptotic perspective. From this view, we aim to develop a method for estimating forces that has the potential to asymptotically converge to the applied force. To achieve this goal, we will compare the excited system to an unexcited estimate system under the assumptions that the system parameters are known and that sampling is continuous in time. Under these assumptions, and concepts from the following subject areas:

- linear dynamical systems theory,
- control theory,
- equilibrium analysis of linear dynamical systems,
- asymptotic approximation analysis,

we will derive a relationship between the excited system, the unexcited system, and the system input. As we are interested in the development and proof of the concepts, we begin the development of this work under ideal circumstances where measurement noise is neglected, and extend these concepts to a real model, which includes measurement noise. Additional details are provided in [18], and while the focus of our applications in this paper are limited to the recovery of forces on mechanical systems, it is noted that the method is applicable to any linear system that satisfies the conditions discussed in this method.

In this paper, the preliminary concepts developed for this analysis are summarized in Section 2. The derivation and significant results of the estimation method for an ideal model are discussed in Section 3. A simple example is developed in Section 4, and the application of the method to the estimation of distributed loads on continuous systems by using modal analysis is shown in Section 5. The concepts of the proposed method are applied to a real model in Section 6. Additional discussion and future work is provided in Section 7.

2. Preliminaries

The development of the proposed estimation method utilizes concepts from multiple disciplines, but while these methods are critical to the development of the method, linear dynamical systems analysis is the starting point for the development. Throughout this work, we use bold face variables to denote matrices and underlined variables denote vectors.

2.1. Equilibrium analysis of linear dynamical systems

The development of the proposed force reconstruction method begins with an analysis of the equilibrium of systems of the form:

$$\dot{\underline{x}} = \boldsymbol{A}\underline{x} + \underline{Q}(t), \tag{1}$$

where $\underline{Q}(t)$ is a time-dependent vector input, $\underline{x}$ is an $n \times 1$ state vector, and $\boldsymbol{A}$ is an invertible, well-conditioned matrix. Our aim is to transform Eq. (1) to a simplified equation of the form:

$$\dot{\underline{z}} = \boldsymbol{A}\underline{z}, \tag{2}$$