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Wavelet-based spectral finite element dynamic analysis for an axially moving Timoshenko beam

Ali Mokhtari, Hamid Reza Mirdamadi *, Mostafa Ghayour

Department of Mechanical Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran

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A B S T R A C T

In this article, wavelet-based spectral finite element (WSFE) model is formulated for time domain and wave domain dynamic analysis of an axially moving Timoshenko beam subjected to axial pretension. The formulation is similar to conventional FFT-based spectral finite element (SFE) model except that Daubechies wavelet basis functions are used for temporal discretization of the governing partial differential equations into a set of ordinary differential equations. The localized nature of Daubechies wavelet basis functions helps to rule out problems of SFE model due to periodicity assumption, especially during inverse Fourier transformation and back to time domain. The high accuracy of WSFE model is then evaluated by comparing its results with those of conventional finite element and SFE results. The effects of moving beam speed and axial tensile force on vibration and wave characteristics, and static and dynamic stabilities of moving beam are investigated.

1. Introduction

The dynamic behavior of axially moving beams is a subject of technological interest since many such materials are observed in manufacturing industries; for example, thread-lines in the fabric industry, rolled steel beams, chain and belt drives, high-speed paper and magnetic tapes, band saw blades, aerial cable tramways, and the like. The lateral vibration of such systems is important to investigate. Beyond a critical moving speed, the axially moving structure may experience a harsh vibration or structural instability resulting in structural failure. Hence, it is essential to predict exactly the dynamic characteristics of such axially moving structures to execute safe, reliable, and successful designs.

Recent developments in research on axially moving structures were reviewed by Wickert and Mote [1], Pellicano and Vestrani [2], and Marynowski and Kapitaniak [3]. The solutions of equations of motion for moving structures were obtained by various solution techniques, including the Galerkin's [2,4,5], assumed modes [6], finite element (FE) [7,8], Green's function [9], transfer function [10], perturbation [11], the Laplace transform [12], artificial parameter [13], complex mode [14], Hilbert-Huang transform (HHT) [15], pseudo-modal parameters [16], and the FFT-based spectral finite element (SFE) methods [17,18].

In the literature [19–21], it is well recognized that the wavelet-based spectral finite element (WSFE) model is an exact solution method for dynamic analysis of structures. FE method is probably the most popular in many areas of engineering. This method might provide accurate dynamic characteristics of a structure if the wavelength would be large compared to mesh size. However, the FE results become increasingly inaccurate as the frequency increases. FE formulation for wave

* Corresponding author.
E-mail addresses: ali.mokhtari@me.iut.ac.ir (A. Mokhtari), hrmirdamadi@cc.iut.ac.ir (H.R. Mirdamadi), ghayour@cc.iut.ac.ir (M. Ghayour).

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Nomenclature

c	constant transport speed [m/s]
c_D	divergence speed [m/s]
c_F	flutter speed [m/s]
EI	flexural rigidity [Nm^2]
$f(x, t)$	excitation force [N]
f_{nyq}	Nyquist frequency [Hz]
$\{\hat{F}^g\}$	global WSFE nodal forces [N, Nm]
k	WSFE wavenumber [rad/m]
$\underline{k_{EB}}$	wavenumber for the Euler-Bernoulli beam theory [$\sqrt{\text{rad}}$/m]
$\underline{k_G}$	pseudo wavenumber resulting from Timoshenko beam theory [$\sqrt{\text{rad/m}}$]
$[\hat{K}^g]$	global dynamic stiffness matrix
L	span between two end supports [m]
L^e	length of an element [m]
$M(x, t)$	bending moment [Nm]
n	number of sampling points
N	order of the Daubechies wavelet
N_x	constant axial pretension [N]
$Q(x, t)$	shear force [N]
r_g	radius of gyration of beam cross section [m]
r_m	shear-to-flexural modulus ratio [1/m^2]
r_N	tensile force-to- flexural rigidity ratio [1/m^2]
$\mathbf{\Gamma}^1$	first-order connection coefficient matrix (time domain)
$\mathbf{\Gamma}^2$	second-order connection coefficient matrix (time domain)
$i\gamma$	eigenvalues of $\mathbf{\Gamma}^1$ [rad/s]
κGA	shear rigidity [N]
$\mathbf{\Lambda}^1$	first-order connection coefficient matrix (wave domain)
$\mathbf{\Lambda}^2$	second-order connection coefficient matrix (wave domain)
$i\lambda$	eigenvalues of $\mathbf{\Lambda}^1$ [rad/s]
ρA	mass per length of beam [kg/m]
ρI	mass moment of inertia per length [kg m]
$\varphi(\tau - k)$	Daubechies scaling function at an arbitrary scale
$\Omega^1_{j-k}, \Omega^2_{j-k}$	connection coefficients
ω_c	cut-off frequency [rad/s]
FE	finite element method
SFE	spectral finite element method
$WSFE$	wavelet-based spectral finite element method

propagation problems needs a large number of partitions for system to reach accurate results, especially, at higher frequencies. These problems are usually solved by SFE model. SFE method [22] transforms the governing partial differential equations (PDEs) of motion to a set of ordinary differential equations (ODEs) by FFT. These 1D (space-dependent) ODEs are solved exactly, which are then used as dynamic shape functions for SFE formulation. WSFE formulation is very similar to SFE formulation, except that Daubechies wavelet basis functions are used for transformation of governing PDEs. The resulting ODEs are coupled unlike those in SFE model, which can be decoupled by using a wavelet-dependent eigenvalue problem. The decoupled ODEs are then solved similarly as in SFE model. When a finite-dimensional structure is weakly damped, it's dynamic response takes a very long time to diminish. If the time window cannot be taken larger than that long decay time, some errors due to periodicity assumption of solutions will appear in the SFE model processing. A potential remedy to reduce these errors is to use artificial damping [22]. In addition, for such methods, the time window should be taken large enough. The time window is dependent on the value of damping and the dimensions of structure. On the contrary, it requires to be longer for weakly damped and shorter dimension structures. WSFE model, formulated by applying non-periodic boundary condition assumptions [20] are entirely independent of time domain deficiencies previously mentioned such as lack of damping, structures having short dimensions, and small time windows. By using this type of boundary conditions, exactness of results could be free from those deficiencies. These assumptions lead to the direct use of WSFE model for time domain analysis unlike SFE model. Periodic boundary condition-based WSFE formulation [23–25] can extract frequency dependent wave characteristics, like wavenumbers, directly.

Recently, Lee et al. [18] have derived the SFE model for axially moving Timoshenko beam to obtain an exact dynamic analysis. However, to the best of authors' knowledge, the WSFE model has not yet been introduced in the literature for axially moving beam structures. Thus, the purposes of this article are: