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A new Gibbs sampling based algorithm for Bayesian model updating with incomplete complex modal data

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ABSTRACT

Model updating using measured system dynamic response has a wide range of applications in system response evaluation and control, health monitoring, or reliability and risk assessment. In this paper, we are interested in model updating of a linear dynamic system with non-classical damping based on incomplete modal data including modal frequencies, damping ratios and partial complex mode shapes of some of the dominant modes. In the proposed algorithm, the identification model is based on a linear structural model where the mass and stiffness matrix are represented as a linear sum of contribution of the corresponding mass and stiffness matrices from the individual prescribed substructures, and the damping matrix is represented as a sum of individual substructures in the case of viscous damping, in terms of mass and stiffness matrices in the case of Rayleigh damping or a combination of the former. To quantify the uncertainties and plausibility of the model parameters, a Bayesian approach is developed. A new Gibbs-sampling based algorithm is proposed that allows for an efficient update of the probability distribution of the model parameters. In addition to the model parameters, the probability distribution of complete mode shapes is also updated. Convergence issues and numerical issues arising in the case of high-dimensionality of the problem are addressed and solutions to tackle these problems are proposed. The effectiveness and efficiency of the proposed method are illustrated by numerical examples with complex modes.

1. Introduction

The need of model updating is usually motivated by the desire to improve the accuracy of prediction of the system response and control [1,2], health monitoring [3,4], or reliability and risk assessment [5,6]. There always exist modeling errors and uncertainties associated with the process of constructing a mathematical model of a system arising either because of incomplete knowledge or simplifying assumptions made during the modeling of the physical problem. These uncertainties in the modeling process can cause the predicted system response to be different from the true system response. If experimental data measured from the system are available, then these data can be used to update the uncertainties in the model parameters.

The usual approach to update a linear dynamic system model is to first identify its modal properties (especially when the data are obtained during ambient vibration) and then use these to update the modeling parameters. There are several ambient or forced vibration based modal identification techniques available [7–12] that provide optimal estimates of the modal

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parameters. Probabilistic model updating techniques, particularly the Bayesian approach, provide estimates of the optimal parameters along with their probability density function (PDF) that can be used to provide a comprehensive quantification of the uncertainty. Several researchers [13–23] presented works on the updating of the Finite element models based on experimental modal data. However, there are relatively few papers in model updating literature in which probabilistic model updating is considered [16–23]. Special attention has been paid to stochastic simulation methods especially MCMC methods that allow generating samples which are distributed according to the posterior PDF without the need of evaluating the normalizing constant in the Bayes' Theorem. Zhang et al. [24] proposed Delayed Rejection Adaptive Metropolis algorithm, and Beck et al. [25] proposed a MCMC (Markov Chain Monte Carlo) method based on Metropolis-Hastings algorithm and simulated annealing for robust system identification. Both approaches experience difficulty in the case of a large number of uncertain model parameters because they require a global proposal distribution. Cheung et al. [26] proposed Hamiltonian Markov chain method with improvements, and Boulkaibet et al. [16] proposed Shadow Hybrid Monte Carlo algorithm to solve higher-dimensional Bayesian model updating problems. However, the efficiency of these algorithms is affected by the complexity of the systems because of the need to solve the eigenvalue problem corresponding to the linear structural dynamic model. Ching et al. [27] proposed TMCMC, and Betz et al. [28] proposed improved TMCMC for Bayesian updating and Bayesian model class selection. However, this approach may experience difficulty in a case with a large number of uncertain model parameters or while updating a complex FE model. Straub et al. [29] presented reliability based method and Au et al. [30] presented subset simulation based approach for uncertainty quantification using Bayesian inference. Their approaches are applicable regardless of the number of uncertain model parameters. However, they become computationally inefficient with increasing complexity of the model. Ching et al. [19] proposed a new Gibbs sampler for model updating of linear dynamic systems based on modal data, and Bansal [31] extended the approach for the case where the modal data are obtained using multiple experimental setup. However, in their algorithms, complex modal data and parameters defining the damping are not considered.

The work presented in this paper is an extension of the work presented by Ching et al. [19]. In this paper, a stochastic simulation algorithm based on Gibbs sampler is presented for Bayesian model updating of a linear dynamic system with non-classical damping based on incomplete complex modal data, namely modal frequencies, damping ratios and partial complex mode shapes of some of the dominant modes. Mathematical formulation for obtaining the conditional distributions required for the application of Gibbs sampler is included. In the proposed algorithm, the identification model is based on a linear structural dynamic model where the mass and stiffness matrix are represented as a linear sum of contribution of the corresponding mass and stiffness matrices from the individual prescribed substructures, and the damping matrix is represented as a sum of individual substructures in the case of viscous damping, in terms of mass and stiffness matrices in the case of Rayleigh damping or a combination of the former. The use of Gibbs sampler makes the proposed approach robust to the number of uncertain parameters involved in the problem. Mode shapes are explicitly considered as uncertain variables that eliminate the need to solve the eigenvalue problem corresponding to the structural dynamic model, which is the most computation demanding part of the model updating problem. An efficient iterative procedure that involves solving of coupled linear regression problems is proposed. Convergence issues and numerical issues arising in the case of high-dimensionality of the problem are addressed and solutions to tackle these problems are proposed. The proposed method is robust to the dimension of the problem. Finally, to demonstrate the effectiveness and accuracy of the proposed method, two numerical examples are presented.

2. Bayesian model updating

A Bayesian model updating approach provides a robust and rigorous framework to characterize and quantify modeling uncertainties. Given the data D and the prior PDF $p(\boldsymbol{\theta})$ of the uncertain system parameters $\boldsymbol{\theta}$, by applying the Bayes' theorem, the posterior PDF $p(\boldsymbol{\theta}|D)$ can be written as:

$$p(\boldsymbol{\theta}|D) = \frac{p(D|\boldsymbol{\theta})p(\boldsymbol{\theta})}{\int p(D|\boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}} \tag{1}$$

As the integral in the denominator of Eq. (1) is often not known explicitly a priori, $p(\boldsymbol{\theta}|D)$ is only known up to a normalizing constant. Based on the topology $p(\boldsymbol{\theta}|D)$ in the parameter space, $\boldsymbol{\theta}$ that maximizes $p(\boldsymbol{\theta}|D)$ which implies maximization of $p(D|\boldsymbol{\theta})p(\boldsymbol{\theta})$ can be classified into three different categories [17]: globally identifiable (unique optimal estimate), locally identifiable (finite number of optimal estimates) and unidentifiable (continuum of optimal estimates). Beck et al. [17] adopted Laplace's method of asymptotic approximation which requires a non-convex optimization [32] to obtain the posterior PDF of the model parameters. However, the accuracy of such an approximation is questionable when either the amount of data is not sufficiently large or the chosen class of models turns out to be unidentifiable based on the available data. Also, the approach is computationally challenging, especially in a high-dimensional parameter space or when the model class is not globally identifiable. To avoid these limitations, in recent years focus has shifted to stochastic simulation methods for Bayesian updating especially MCMC methods.

Assume that $\boldsymbol{\theta}$ is partitioned into G groups of uncertain parameter vectors, i.e. $\boldsymbol{\theta} = [\boldsymbol{\theta}_j : j = 1, \ldots, G]$. The Gibbs Sampler [33] is one type of MCMC algorithms that allow sampling from an arbitrary multivariate PDF if sampling according to the PDF of each group of uncertain parameters conditioned on all the others groups is possible. The advantage of Gibbs Sampling meth-