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Fast computation of the spectral correlation

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ABSTRACT

Although the Spectral Correlation is one of the most versatile spectral tools to analyze cyclostationary signals (i.e. signals comprising hidden periodicities or repetitive patterns), its use in condition monitoring has so far been hindered by its high computational cost. The Cyclic Modulation Spectrum (the Fourier transform of the spectrogram) stands as a much faster alternative, yet it suffers from the uncertainty principle and is thus limited to detect relatively slow periodic modulations. This paper fixes the situation by proposing a new fast estimator of the spectral correlation, the *Fast Spectral Correlation*, based on the short-time Fourier transform (STFT). It proceeds from the property that, for a cyclostationary signal, the STFT evidences periodic flows of energy *in* and *across* its frequency bins. The Fourier transform of the interactions of the STFT coefficients then returns a quantity which scans the Spectral Correlation along its cyclic frequency axis. The gain in computational cost as compared to the conventional estimator is like the ratio of the signal length to the STFT window length and can therefore be considerable. The validity of the proposed estimator is demonstrated on non trivial vibration signals (very weak bearing signatures and speed varying cases) and its computational advantage is used to compute a new quantity, the *Enhanced Envelope Spectrum*.

Conventions

Whereas the SC $S_x(\alpha,f)$ is a theoretical quantity, the ACP $S_x^{ACP}(\alpha,f)$, the CMS $S_x^{CMS}(\alpha,f)$, and the Fast-SC $S_x^{Fast}(\alpha,f)$ are three different estimators of the SC.

The connections between the spectral quantities handled in the paper are schemed in Fig. 1.

Definitions of the SC found in the literature may differ in the measurement units. The definition given here is such that for a signal with measurement units U, the SC has units U^2/Hz. It is a one dimensional density of variable f. As a consequence, the particular case $\alpha=0$ returns the power spectral density, $S_x(0,f)\equiv S_x(f)$. Another definition of the SC is actually as a two dimensional density of variables f and α, with units U^2/Hz^2 [1]. The power spectral density is then evaluated as $S_x(f)=\lim_{B\to 0}\int_{-B}^{B}S_x(\alpha,f)d\alpha$.

1. Introduction

Whether of mechanical or electrical nature, rotating machine signals are perfectly modelled by cyclostationary processes. The reason is that, due to the inherent operation of a machine, signals are produced by some periodic – or cyclic – mechanisms.

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Nomenclature

SC	Spectral Correlation
ACP	Averaged Cyclic Periodogram
CMS	Cyclic Modulation Spectrum
Fast-SC	Fast Spectral Correlation
DFT	Discrete Fourier Transform
FAM	FFT Accumulation Method
FFT	Fast Fourier Transform
STFT	Short-Time Fourier Transform
SES	Squared Envelope Spectrum
EES	Enhanced Envelope Spectrum
OF	Order-Frequency
CPU	Central Processing Unit
$x(t_n)$	signal of interest
$w[n]$	data window (function of time index n)
$X_w(i,f)$	Gabor coefficient at time index i and frequency f
$X_{STFT}(i,f)$	STFT coefficient at time index i and frequency f
L	signal length
N_w	window length in STFT
N_0	central time index of window
R	block shift in STFT
K	total number of blocks used in spectral estimates
F_s	sampling frequency
t_n	n-th discrete time instant (in s)
τ	time-lag (in s)
T	cyclic period of a cyclostationary signal (in s)
α	cyclic (or modulation) frequency (in Hz)
α_{max}	maximum scrutinizable cyclic frequency (in Hz)
f	spectral (or carrier) frequency (in Hz)
f_k	k-th discrete frequency (in Hz)
$\Delta\alpha$	cyclic frequency resolution in α (in Hz)
Δf	frequency resolution in f (in Hz)
p	index of STFT frequency closest to a given cyclic frequency α
P	index of STFT frequency closest to α_{max}
$R_x(t_n,\tau)$	instantaneous autocorrelation function of signal x
$R_w(\alpha)$	discrete Fourier transform of $\lvert w[n]\rvert^2$
$S_x(\alpha,f)$	Spectral Correlation of signal x
$\gamma_x(\alpha,f)$	Spectral Coherence of signal x
$S_x^{ACP}(\alpha,f)$	Averaged Cyclic Periodogram of signal x
$S_x^{CMS}(\alpha,f)$	Cyclic Modulation Spectrum of signal x
$S_x(\alpha,f;p)$	Scanning Spectral Correlation of signal x
$S_x^{Fast}(\alpha,f)$	Fast Spectral Correlation of signal x
$\gamma_x^{Fast}(\alpha,f)$	Fast Spectral Coherence of signal x
$S_x^{SES}(\alpha)$	Squared Envelope Spectrum of signal x
$S_x^{EES}(\alpha)$	Enhanced Envelope Spectrum of signal x
$\mathcal{C}_{CMS}$	computational complexity of Cyclic Modulation Spectrum
$\mathcal{C}_{ACP}$	computational complexity of Averaged Cyclic Periodogram
$\mathcal{C}_{Fast}$	computational complexity of Fast Spectral Correlation

The cyclostationary class defines processes whose statistics are periodic. It encompasses most of the processes usually encountered in machines as particular cases, be they deterministic or random, e.g. periodic signals, stationary signals, periodically-modulated signals, repetitive transients, etc. This makes cyclostationarity a preferred framework in vibration-based condition monitoring. Because of its ability to perfectly describe the statistical behavior of faults in the form of symptomatic modulations or repetition of transients, it provides optimal tools for their detection, their identification, and possibly their quantification. One central tool for the "cyclic spectral analysis" of machine signals is the Spectral Correlation (SC) which displays at once, in the form of a bi-spectral map, the whole structure of modulations and carriers in a signal [2–6]. Although the demonstration of the capabilities of cyclic spectral analysis in condition monitoring has been undertaken in several research works [7–13], its practice is still not as systematic as it deserves. Many methods are constantly published