Mechanical Systems and Signal Processing 98 (2018) 50–62

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

# A modal-energy based equivalent lumped model for open cracks

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ARTICLE INFO

*Article history:*
Received 5 July 2016
Received in revised form 20 April 2017
Accepted 25 April 2017
Available online 3 May 2017

*Keywords:*
Open crack
Exact modeling
Equivalent modeling
Natural frequency

ABSTRACT

Two modeling methods are proposed for open notch-like cracks in beam structures. In exact modeling method, the cracked beam is divided into 3 parts – two beams and one crack parts – and its natural frequency analysis is done by assuming that shear beam theory governs the dynamic behavior of the structure. In equivalent modeling method, an equivalent lumped model composed of a set of lateral, torsional and lateral-torsional springs is considered for crack part. The stiffness coefficients of the equivalent lumped model are obtained by comparing the two modeling methods in a modal-energy based approach. The accuracy of the proposed modeling methods are verified by numerous numerical and experimental case studies.



## 1. Introduction

There are many mechanical structures containing beam-like members whose modeling, dynamic response analysis and identification demand precise mathematical representations being capable to regenerate dynamical behavior accurately. This is especially more important when cracks initiate or grow in beam-like structures. Cracks change the local stiffness of structures and hence alter their dynamical characteristics such as natural frequencies and mode shapes. Identification of crack location and crack severity by using vibration data has attracted the focus of many papers in the past decades. Two approaches – namely model based and non-model based approaches – have been widely used in literature for crack identification [1]. The common type of vibration data usually used for crack identification contains natural frequencies, mode shapes, curvature (strain) mode shapes and dynamical flexibility [2].

Model-based identification approaches seek to characterize the crack parameters – i.e. location and severity – by minimizing the residual between a mathematical model and experimental data. Although the quality of the experimental data effectively affects the identification results, the successful of model-based identification approaches rely primarily on the precision of the mathematical models used. Mathematical models – to be effectively used in identification procedures – should contain precise representation of the crack in the structure. Many modeling methods have been employed by researchers for considering open cracks in structures. These modeling methods can be categorized into exact and equivalent modeling methods.

In equivalent modeling methods the effect of crack in structure is considers as a linear or torsion spring. Cawley and Adams [3] investigated the axial vibration of uniform bars and suggested that the stiffness reduction due to the presence of a crack in a uniform bar can be modeled as a linear spring. Since then many researcher tried to relate the crack equivalent spring coefficient to crack severity in the structures for single-sided and double-sided cracks. Ju et al. [4] used some assumptions such that small deformation and homogenous and isotropic material and theoretically derived a stiffness function for

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http://dx.doi.org/10.1016/j.ymssp.2017.04.038
0888-3270/

open double sided cracks by using a relationship from fracture mechanics between the strain energy and stress intensity factor. Ostachowicz and Krawczuk [5] proposed stiffness coefficient functions for both single-sided and double-sided open cracks. They found that the stiffness coefficient is a function of crack depth and beam height. Chondros et al. [6] derived a function for the local flexibility of single-sided open cracks in beam structures. Tomasel et al. [7] used the two forms of crack functions proposed in Refs. [5,6] for open single sided cracks and concluded that the best results were achieved by using the one proposed by Chondros et al. [6]. Haisty and Springer [8] developed an expression for the equivalent spring coefficient of a double-sided open crack by adopting a strain energy density function obtained by least-square curve fitting on finite element results. It is worth mentioning that the deviation between different crack equivalent stiffness coefficients described in above, originates from different strain energy density function used by researchers.

In exact modeling methods the effect of crack in beam-like structures is considered as reduction in sectional bending stiffness and sectional mass and the equations governing the dynamic response of cracked beam are solved. Dixit and Hanagud [9] modeled the notch-like cracks in beam structures by considering the stiffness and mass reduction due to crack as explicit functions for sectional bending stiffness and sectional mass of beam by using Heaviside functions. Christides and Barr [10] modeled the effect of crack in rectangular beams as reduction in only cross sectional stiffness by using an exponential function. Cross sectional stiffness and cross sectional mass functions proposed in [9,10] are complex functions. Sinha et al. [11] used a linear varying function for cross sectional stiffness of cracked beams. Law and Lu [12] considered the effect of crack on cross sectional stiffness of a beam by using Dirac delta function.

This paper aims to consider modeling open notch-like cracks with considerable dimensions – i.e. considerable width and depth. The width of notch-like cracks has seldom been the subject of published articles in the literature. Two exact and equivalent modeling approaches are proposed in this paper for cracked beams. In contrast to the exact modeling approaches in the literature, in this paper the width of crack is incorporated in the governing equations and shear beam theory is employed for free vibration analysis. Moreover, in the equivalent modeling approaches in the literature, the crack is modeled as a single torsion spring. In this paper a combination of three lateral, torsional and lateral-torsional springs is considered as the equivalent crack model. A modal-energy based approach is also employed for obtaining the stiffness coefficients of the equivalent crack model.

In fact, the exact modeling method proposed in this paper paves a foundation for obtaining an equivalent model for open cracks which is: (1) much more accurate than the equivalent models in the literature, (2) is more realistic and its stiffness coefficients are functions of both width and depth of crack and (3) is simpler than the exact modeling method and makes vibration analysis of the structures with multiple cracks much easier.

The modeling methods proposed in this paper show promising results compared to the exact and equivalent modeling methods existing in the literature. The good level of accuracy of the modeling methods in this paper owes to considering a more realistic open notch-like crack in governing equations of exact modeling approach and improvement made in crack model in equivalent modeling approach.

The remaining of this paper is organized as follows: in Section 2 dynamic modeling of a cracked beam structure by assuming the crack as a reduction in cross section with considerable dimensions are considered. In Sections 3 and 4, a new equivalent lumped model for open cracks is proposed and the approach for calculating its stiffness coefficients is introduced. Section 5 considers the free vibration analysis of cracked beams with crack models as a torsion spring. Numerical and experimental verifications are considers in Section 6. Conclusions are drawn and references are provided at the end of paper.

## 2. Exact modeling method

A constant rectangular cross-section beam-like structure containing an open notch like single or double-sided crack in position $L_1$ is considered as is shown in Fig. 1. The width and depth of crack are respectively $a$ and $w$ where $a = h - h_c$. Since

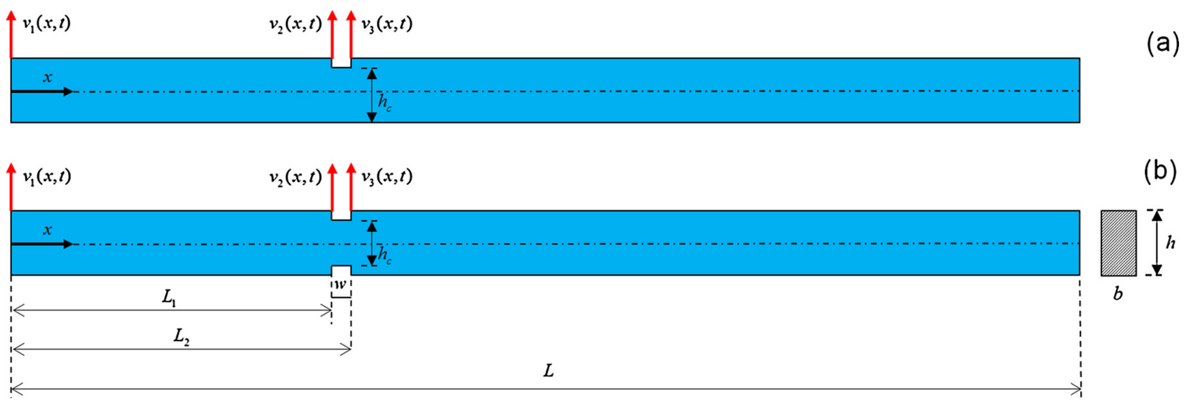


**Fig. 1.** Beam-like structure with open cracks: single-sided (a) and double-sided (b).