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Coupled oscillators in identification of nonlinear damping of a real parametric pendulum

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ABSTRACT

A damped parametric pendulum with friction is identified twice by means of its precise and imprecise mathematical model. A laboratory test stand designed for experimental investigations of nonlinear effects determined by a viscous resistance and the stick-slip phenomenon serves as the model mechanical system. An influence of accurateness of mathematical modeling on the time variability of the nonlinear damping coefficient of the oscillator is proved. A free decay response of a precisely and imprecisely modeled physical pendulum is dependent on two different time-varying coefficients of damping. The coefficients of the analyzed parametric oscillator are identified with the use of a new semi-empirical method based on a coupled oscillators approach, utilizing the fractional order derivative of the discrete measurement series treated as an input to the numerical model. Results of application of the proposed method of identification of the nonlinear coefficients of the damped parametric oscillator have been illustrated and extensively discussed.

1. Introduction

1.1. Literature overview

Nonlinear dynamical models are commonly applied to assess the properties and to predict the performance of miscellaneous engineering objects.

Dynamic modeling of a mechanical system with nonlinear strain-frequency-dependent damping is carried out in [1]. Nonlinear damping information on a damping alloy specimen has been extracted from the free decay signal by means of the moving autoregressive model method. The viscoelastic theory is introduced to describe the strain-frequency-dependent characteristics of damping more accurately, a viscoelastic three-parameter structural damping constitution model is developed, the parameters of which are identified from the test data by means of an optimization algorithm. The finite element dynamic equations for strain-frequency-dependent damping are derived through the established three parameters constitution. The established finite element dynamic equations are assembled into the system dynamic equations of an elastic linkage mechanism by means of the kineto-elastodynamic theory, and a closed-form numerical algorithm is constructed in order to solve the high-order differential equations with time-varying coefficients.

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In the design process, it is essential to use accurate numerical simulation tools to predict the complex aero-hydro-servo-elastic response of a floating wind turbine [2]. The cited paper focuses on the use of the open-water test data of the SWAY prototype wind turbine to calibrate a floating offshore wind turbine numerical model for future validation efforts. After turbine deployment and installation of the NREL instrumentation, five free decay tests were conducted on the SWAY prototype by displacing the system and allowing it to return to equilibrium. The inability to model frictional damping in the universal joints of the system contributes to discrepancies between measured and simulated results. The inability to model frictional damping in the universal joints in the tension rod became significant in affecting the overall motion of the system.

In offshore engineering and naval architecture, it is a common practice to determine damping coefficients, both linear and nonlinear, from free decay tests [3]. For example, damping coefficients of floating structures are often obtained in this manner in ocean engineering [4]. The paper presents some work on the determination of nonlinear damping coefficients for flap-type oscillating wave surge converters from free decay tests. Simulations of the free decay tests in the computational fluid dynamics are presented as well as their validation against experimental results is performed. Analysis of the obtained data reveals that linear damping, as commonly used in time domain models, is unable to accurately model the occurring damping over the whole regime of rotation amplitudes. The authors concluded that a hyperbolic function is most suitable to express the instantaneous damping ratio over the rotation amplitude.

A nonlinear inverse method of nonparametric identification is proposed in [5] for both recovering the full nonlinear damping and restoring functions in a harmonic forced nonlinear oscillator. For this purpose, a proper inverse problem and its mathematical formalism are developed by introducing the intersection and zero-crossing times with respect to motion response, based on the acceleration response measurements. The identification does not depend on the usual regularizations, which is generally essential to the usual (ill-posed) inverse problems arising in mathematical science and engineering. As a model equation, a highly nonlinear system is examined for the workability of the inverse method proposed through numerical experiments.

In calculations of vibration serviceability, the damping value of the structural systems is a critical parameter. The amplitude-dependent damping behavior of the laboratory footbridge investigated in [6] is subjected to different amplitudes of sinusoidal excitation. The amplitude-dependent damping ratio values obtained from effective mass calculations proved to be correct with the FE model acceleration predictions. The FE model predictions successfully matched the test results with the nonlinear characteristic introduced for modal damping.

An equivalent linearization method is adopted in [7] to model the added damping and stiffness effect via free decay response in zero wind speed condition. It is found that the nonlinear characteristics of added damping effect are rather obvious while that of added stiffness effects is considerably weak.

These examples and many other problems reported in [8–11] after measurement of stiffness or damping confirm the necessity of deeper investigations on the techniques related to identification of parameters of vibrating bodies. It becomes especially important for the investigation of a vibrating physical systems, among others, viscous, Coulomb, Rayleigh or modal damping is necessary to be determined [8,12–15].

Various functions of nonlinear damping are incorporated in more or less advanced mathematical models of dynamical systems. Among these applications, one can distinguish: a nonlinear damped wave equation with Dirichlet boundary condition [16]; a single-degree-of-freedom (SDOF) passive vibration isolation system with geometrically nonlinear damping [17]; dynamical model of bearing supported with a nonlinear damping force and a linear elastic restoring force [18]; clamped beam with tip mass, coupled to nonlinear spring and damper [19]; an extension of Caughey's linear damping models to a nonlinear class [20]; generalized Rayleigh's dissipation function, including isotropic, nonlinear damping in the Euler ”Lagrange equations [21]; a novel causal damping model capable of expanding the constant frequency area [15].

Identification of damping or modal damping ratio in 1-DOF systems was investigated also in other literature. The extraction of information about damping from hysteresis loops, Nyquist plots, multi-frequency excitation method [22] or various free decay tests [23–27] and even from wavelets [28–31] should be mentioned here.

A purpose of this article is also to make a step towards inclusion of measurement data, as the input signal, into a set of coupled differential equations to identify the nonlinear time-varying parameters of real dynamical systems.

In what follows we briefly discuss a few earlier investigations on identification of damping, putting emphasis on the need of experimental research to achieve reliable and validated results.

The problem aimed at modeling of a nonlinear damping has been considered by Yuan [1], but this research was focused on a particular mechanical object, i.e., an elastic mechanism.

Significance of numerical simulation in the prediction of dynamic responses is illustrated and discussed in Ref. [2]. The mentioned work strengthens the need of using the experimental data, but it does not fully solve the problem of identification of frictional damping. This observation is confirmed by discrepancies between both the measured and simulated system responses. Measurement data coming from real tests is required for validations based on numerical models, and the authors have partially solved this main problem.

Damped vibrations determine dissipation of energy in dynamical systems. Therefore, determination of their parameters is important, for instance, to observe changes in properties of the systems without the necessity of repeating of time-consuming tests. This remark seems to be obvious, but the determination of damping properties is useful not only to get a precise mathematical model, but also for the faithful numerical simulations reflecting our reality.

Some observations and conclusions similar to that aforementioned can be drawn according to methodology and results presented in the works [3,4].