Mechanical Systems and Signal Processing 98 (2018) 190–208

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Construction and identification of a *D-Vine* model applied to the probability distribution of modal parameters in structural dynamics

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A R T I C L E   I N F O

Article history:
Received 3 October 2016
Received in revised form 24 March 2017
Accepted 18 April 2017
Available online 8 May 2017

Keywords:
Probabilistic modal superposition
Random transfer functions
Copula theory
Pair-copula construction
D-Vine

A B S T R A C T

This study investigates the construction and identification of the probability distribution of random modal parameters (natural frequencies and effective parameters) in structural dynamics. As these parameters present various types of dependence structures, the retained approach is based on pair copula construction (PCC). A literature review leads us to choose a *D-Vine* model for the construction of modal parameters probability distributions. Identification of this model is based on likelihood maximization which makes it sensitive to the dimension of the distribution, namely the number of considered modes in our context. To this respect, a mode selection preprocessing step is proposed. It allows the selection of the relevant random modes for a given transfer function. The second point, addressed in this study, concerns the choice of the *D-Vine* model. Indeed, *D-Vine* model is not uniquely defined. Two strategies are proposed and compared. The first one is based on the context of the study whereas the second one is purely based on statistical considerations. Finally, the proposed approaches are numerically studied and compared with respect to their capabilities, first in the identification of the probability distribution of random modal parameters and second in the estimation of the 99% quantiles of some transfer functions.

1. Introduction

Probabilistic approaches are nowadays largely used in structural mechanics to handle quantification and propagation of uncertainties through a mechanical model. A global methodology, introduced for example in [1], is now well established and gives the framework to deal with parametric uncertainties affecting the inputs of a mechanical model. A critical point in this methodology concerns propagation of randomness from the model inputs to its outputs.

In the field of deterministic linear structural dynamics, a common way to solve motion equation is to use modal superposition method. It implies the resolution of an eigenvalue problem and the construction of transfer functions by linear combinations, involving both the eigenvalues and some terms of the eigenvectors, the so-called modal parameters.

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http://dx.doi.org/10.1016/j.ymssp.2017.04.024

In this context, probabilistic approach leads to a random eigenvalue problem. Determination of the probability distributions of eigenvalues and eigenvectors of random matrices is addressed by random matrix theory [2,3]. Nevertheless, this theory mainly deals with random matrices defined by some specific probability distributions. For example, Gaussian ensembles have been widely studied and many theoretical results describe the statistics of their eigenvalues and eigenvectors. Unfortunately, random matrices encountered in engineering problems do not belong to these sets. In the field of linear stochastic dynamical system, random matrix theory has been applied by [4] where Wishart random matrix model is studied and by [5,6] where a non parametric random matrix model is constructed by the use of maximum entropy principle.

A large class of approximation methods has also been successfully applied to this problem. One of the first is based on Taylor series or perturbation method [7]. The main idea is to use first or second order Taylor series expansions of the eigenvalues and eigenvectors in terms of the random inputs. Polynomial chaos expansion [8,9], which is based on the decomposition of second order random variables into a polynomial expansion, has also been applied to random eigenvalue problems. For example in [10], a comparison between polynomial chaos representation and perturbation method is provided, concluding that the former performed better than the latter except if the variability of random inputs is small. In [11] an asymptotic approach is proposed which leads to an expression of the joint probability density function of the eigenvalues. According to the presented results, this approximation approach outperforms second order perturbation approach. However, the random effective parameters are not considered in this work. Polynomial chaos expansion has also been applied to various structural dynamics application (see [12] for example). Another method, based on a dimensional decomposition of a random variable [13], is applied to the random eigenvalues problem in [14]. Finally, metamodels approaches can also be applied such as surface response [15]. It should also be noted that approximation methods have been applied to the resolution of random vibration analysis out of the framework of modal superposition. For example, proper generalized decomposition approximation is used in [16] for the problem of uncertainty in structural dynamics, leading to a reduced order model based on deterministic basis. As these methods are based on functional approximations, their computational cost increase with the number of random inputs which is their main drawback.

One can also note that simulation based methods are relevant in this context as they are not sensitive to the stochastic dimensionality of the problem and are usually easy to implement on industrial models. In theory, direct Monte Carlo Simulation (MCS) method could be used to solve any random eigenvalues problem. But the numerical cost, it involves, becomes unrealistic when one deals with large industrial models. Consequently, research activities on this field are mainly devoted to increasing the computation efficiency of the resolution of one realization of the random eigenvalues problem. For example, [17,18] propose to use subspace iteration method and selection of *smart* starting vector to increase the efficiency of the eigenvalues problem solver.

The method, we propose in the following, is based on the identification of the probability distribution of the modal parameters involved in a given random transfer function. It should be noted that the identified probability distribution can then be used to perform various statistical analysis of the transfer function at low computational cost (as the expression of the transfer function with respect to the modal parameters is analytic). Especially, one can easily study the random transfer function for a wide range of input excitation (external force or imposed motion of the junction). The keystone is then the construction and identification of large dimension probability distribution. A relevant way to achieve this goal is proposed in [19,20]. This method, called pair copula construction (PCC) of joint probability distribution, is based on identification of bivariate copula [21]. We propose to apply this methodology to the identification of the probability distribution of modal parameters. Objectives of this study are mainly to adapt this identification method to the context of structural dynamics and to present the interests of this approach.

The next section of this paper gives some details about the problem that we are interested in: Identification of the modal parameters probability distribution and computation of random transfer function by modal superposition. The third section recalls basics on PCC. The fourth shows how to use PCC in identification of modal parameters probability distribution and highlights specific difficulties. Section five gives applications of the proposed methodology. Finally a concluding part recalls and discusses the results.

2. Modal superposition method and random eigenvalue problem

2.1. Deterministic case

We consider the vibratory motion of an elastic body excited either by an external force or by a junction displacement, in the low frequency range. The structure is assumed to be discretized by the finite element method. Using the notations introduced in [22], the motion equation reads

$$\left(-\omega^2\begin{bmatrix}\underline{\mathbf{M}}_{ii} & \underline{\mathbf{M}}_{il}\\ \underline{\mathbf{M}}_{li} & \underline{\mathbf{M}}_{ll}\end{bmatrix}+\mathrm{j}\omega\begin{bmatrix}\underline{\mathbf{C}}_{ii} & \underline{\mathbf{C}}_{il}\\ \underline{\mathbf{C}}_{li} & \underline{\mathbf{C}}_{ll}\end{bmatrix}+\begin{bmatrix}\underline{\mathbf{K}}_{ii} & \underline{\mathbf{K}}_{il}\\ \underline{\mathbf{K}}_{li} & \underline{\mathbf{K}}_{ll}\end{bmatrix}\right)\begin{Bmatrix}\underline{\mathbf{X}}_i(\omega)\\ \underline{\mathbf{X}}_l(\omega)\end{Bmatrix}=\begin{Bmatrix}\underline{\mathbf{F}}_i(\omega)\\ \underline{\mathbf{F}}_l(\omega)\end{Bmatrix} \tag{1}$$

where $\mathrm{j}^2=-1$. A matrix $\mathbf{X}_{il}$ denotes a matrix of size (n_i, n_l), where n_i is the number of internal DOF (degrees of freedom), indexed by i, and n_l the number of junction DOF, indexed by l. Moreover, with these notations, $\mathbf{X}_{li}$ is the transposed matrix of $\mathbf{X}_{il}$ and $\mathbf{X}_{ii}$ is symmetric. $[\underline{\mathbf{M}}]$, $[\underline{\mathbf{K}}]$ and $[\underline{\mathbf{C}}]$ are respectively the mass, stiffness and damping matrices of the structure, calculated for the mean values of input parameters. $\underline{\mathbf{X}}_i$ is the vector of internal DOF, $\underline{\mathbf{X}}_l$ the vector of junction DOF, $\underline{\mathbf{F}}_i$ the vector of