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# Nonlinear state-space modelling of the kinematics of an oscillating circular cylinder in a fluid flow

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ABSTRACT

The flow-induced vibration of bluff bodies is an important problem of many marine, civil, or mechanical engineers. In the design phase of such structures, it is vital to obtain good predictions of the fluid forces acting on the structure. Current methods rely on computational fluid dynamic simulations (CFD), with a too high computational cost to be effectively used in the design phase or for control applications. Alternative methods use heuristic mathematical models of the fluid forces, but these lack the accuracy (they often assume the system to be linear) or flexibility to be useful over a wide operating range.

In this work we show that it is possible to build an accurate, flexible and low-computational-cost mathematical model using nonlinear system identification techniques. This model is data driven: it is trained over a user-defined region of interest using data obtained from experiments or simulations, or both. Here we use a Van der Pol oscillator as well as CFD simulations of an oscillating circular cylinder to generate the training data. Then a discrete-time polynomial nonlinear state-space model is fit to the data. This model relates the oscillation of the cylinder to the force that the fluid exerts on the cylinder. The model is finally validated over a wide range of oscillation frequencies and amplitudes, both inside and outside the so-called lock-in region. We show that forces simulated by the model are in good agreement with the data obtained from CFD.



## 1. Introduction

The kinematics of oscillating bluff bodies in a fluid flow have been a hot research topic for decades. Most important are the potentially harmful vortex-induced vibrations (VIV) where the structure is excited by alternating vortex shedding. Classical examples are the vibration of chimney stacks exposed to wind, pipelines on the sea-bed excited by the ocean currents or tubes in heat exchangers [1].

For this kind of fluid-structure interactions, typically an analytical solution cannot be found. However, accurate predictions of the kinematics and the resulting dynamics are vital for instance in the design process, in monitoring applications or for control. To obtain predictions with high fidelity, so far only two possibilities existed: solving the Navier-Stokes equation via computation fluid dynamic (CFD) simulations or performing experiments. Both approaches are time-consuming,

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expensive, and require dedicated lab or computing facilities. These drawbacks make the current methods too expensive for many intended applications, where only limited time and resources might be available.

To tackle the shortcomings of CFD and experiments, low-order models of VIV have been developed [2]. Most models are heuristic, with only a limited range of applicability. Different types exist, the most noteworthy can be classified as force decomposition models [3], single degree-of-freedom (sdof) models [4] or wake oscillator models, where the wake oscillation is generally formulated as a Van der Pol equation [5–7].

Although capable of qualitatively describing the characteristic behaviour of VIV, an efficient and powerful model, flexible enough to span a wide domain in parameter space with a single set of parameters has yet to be obtained. To identify such a model, system identification can be a very powerful tool. In recent years, the application of linear system identification techniques has already shown some promising results, e.g. the best linear approximation in least squares sense [8] or building an input-output relationship using delayed values of the input and the output (ARX model) [9].

Nevertheless, the challenges in modelling the system at hand are substantial. Fluid-structure interactions are, for one, inherently nonlinear [10]. This is strongly pronounced by the fact that VIV is a selfexcited yet self-limited oscillation, resulting in a stable limit cycle [11]. In addition, it has been shown that the vortex shedding behaviour is hysteretic [12]. Nonlinear modelling of VIV is currently a very active research topic. In [13], the auto-regressive moving averaging (ARMA) technique is combined with the modal analysis method. A nonlinear force representation is obtained by including higher harmonic terms resulting from the fluid-structure coupling. The coefficients in the model are determined to provide a best fit to the measured time series in maximum-likelihood sense. Reasonable predictions in terms of maximum amplitude are found, although only a limited amount of validation cases were investigated. In recent work [14], local linear models are used as a surrogate model for aerodynamic loads on wings. A general framework is however not yet in place when it comes to nonlinear identification of complex dynamical systems. A lot can still be gained in terms of quality and applicability of the model but also in terms of insight into the nonlinear behaviour of VIV.

In this work, we propose a novel approach to model flow problems that applies state of the art nonlinear identification techniques. The powerful framework of nonlinear state-space models is explored and put to the test. Nonlinear state-space models have proven to be very rich and flexible, providing solutions to a variety of nonlinear problems also including hysteresis [15,16].

The methodology of this work can be summarised as follows:

1. Use established methods, CFD simulations in this case, although the proposed method is equally suitable for experiments, to study the fluid forces within a user defined domain of parameter space. Then capture these observations under the form of time series.
2. Identify a nonlinear black-box model that is able to reproduce the observed phenomenon.
3. Exploit the benefits of the obtained model to simulate, at very low cost, a variety of possible regimes of the system.

The lay-out of the paper is as follows: in Section 2, the simulation set-up is introduced. Section 3 describes the discrete polynomial nonlinear state-space model structure and how such a model is identified. Section 4 consists of two parts. In the first part, the method is applied to a nonlinear oscillator of the Van der Pol type. In the second part the kinematics of an oscillating cylinder is addressed. Section 5 concludes the paper.

## 2. Simulation set-up

VIV are often studied on a 2D circular cylinder, suspended by a spring and a damper and restricted to move in the transverse direction, perpendicular to the incoming flow. Such a set-up is a self-excited oscillator, where the fluid forces generate the displacement. Common practice, however, is the use of imposed (or forced) oscillations [12,17,18] to study the interaction between the flow field and the cylinder. With total control of the displacement one can explore a wide range of frequencies and amplitudes, contrary to the limited response behaviour observed when the mass, stiffness and the damping are fixed to certain values. The set-up of both the free and the imposed oscillations can be seen in Fig. 1.

### 2.1. Input-output description of the system

Since the displacement is imposed, this is logically considered the input to the system. To quantify the behaviour of the wake one could consider any of the classical fluid variables: velocity, vorticity or pressure in some points downstream. Although not all variables are equally informative [19], they remain good candidates to build a model from. More obvious, from an application point of view, is a displacement-force relationship. Moreover, the fluid force on the cylinder can be seen as a combined fluid variable, concentrated in a single observable point, the centre of the cylinder. The system can thus be considered as a single input-single output system. To generalise the findings, both the input and the output are made dimensionless. The input is given by $A/D$ with $A$ the amplitude of the displacement and $D$ the diameter of the cylinder. The output is given by the force coefficient $c_y$,