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A smoothing algorithm for joint input-state estimation in structural dynamics

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ABSTRACT

This paper presents a recursive algorithm where a time delay is considered in the estimation of the forces applied to a structure and the corresponding system states. In particular when the measured response is not collocated with the estimated forces, essential information on the estimated forces and/or system states is contained in the response at L consecutive time steps following the time step where the estimation is performed. The main focus in this paper is on the reduction in estimation uncertainty that can be achieved by so-called smoothing, i.e. by considering a time delay in the estimation. When the calculation of the gain matrices is included in the recursive estimation, the calculation time of the algorithm largely increases with the time delay. It is shown that a prior calculation of the steady-state gain matrices allows for a significant reduction of the calculation time. The presented algorithm is first verified using numerical simulations. Next, a validation is performed using data obtained from a field test on a footbridge.

1. Introduction

For many civil engineering structures, the dynamic loads cannot be directly measured. This is for example the case for wind loads acting on wind turbines or tall buildings, where data on such loads can be of interest for the design of future structures and the validation of load models prescribed by the Eurocodes or other design guidelines. In addition, it is practically and economically infeasible to measure the response of the structure at all locations of interest. For example, direct strain measurements in the tower of an offshore wind turbine below the water surface are difficult due to very harsh conditions. In these cases, inverse techniques can be applied for force and response estimation, hereby combining vibration data from a limited number of sensors with the information obtained from a dynamic model of the structure.

A wide variety of system inversion algorithms for force and state/response estimation has been proposed in the literature, tackling the system inversion problem in the frequency or time domain. Within the time domain approaches, a distinction can be made between deterministic approaches [1–3] and recursive filtering and smoothing approaches [4–6]. Filtering algorithms predict the input and states at time step k from the (known) response measured from time step 0 up to time step k, i.e. no delay is applied in the estimation. Note that only the response at time step k is explicitly used in the estimation of the input and/or states at the same time step k. Through recursion, the estimates at time step k depend on the response at all previous time steps, however. If a delay L is adopted in the estimation, i.e. the response from time step 0 to $k+L$ is used

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to (recursively) estimate the input and states at time step k, with $L > 0$, the estimation algorithm is classified as a smoothing algorithm. Smoothing algorithms can alternatively be classified as a specific type of moving horizon estimation (MHE) algorithms, which use the observations within a predefined time window (k to $k+L$) to estimate unknown variables, e.g. the system states [7]. Whereas MHE algorithms generally use (iterative) optimization to minimize the estimation uncertainty for separate time windows, smoothing algorithms implicitly optimize the solution through the choice of the gain matrices, which depend on the assumed noise statistics. Both filtering and smoothing algorithms are generally less computationally expensive than MHE algorithms, making them particularly suited for continuous health monitoring of structures over their lifetime. Because MHE algorithms are based on iterative optimization, the formulation of the system inversion problem is more general and allows including a priori information on the solution, however.

Recently, much focus has gone to joint input-state estimation, where the forces applied to the structure and the corresponding system states are simultaneously estimated. Joint input-state estimation is mostly performed by use of recursive Kalman filter based techniques [8–13]. The conditions for (instantaneous) system inversion have been extensively documented in the literature [14–16]. Floquet and Barbot [17] showed that these conditions can be relaxed by allowing some delay in the estimation. Recently, Hsieh [18] developed a time delayed joint input-state estimation algorithm as an extension of an existing recursive three-step filtering algorithm [19].

This paper presents a recursive smoothing algorithm where a time delay is considered in joint input-state estimation. The developed approach is similar to that given in [18,20], but with a different filtering structure. The smoothing algorithm can be applied for force identification and response estimation and is an extension of a state-of-the-art filtering algorithm for joint input-state estimation [10,13]. Where recent work on time-delayed system inversion has mainly focused on the relaxation of the invertibility criteria through the introduction of a time delay [18,20], this paper mainly focuses on the reduction in estimation uncertainty. It is shown by numerical simulations that introducing a time delay in the estimation allows to significantly reduce the estimation uncertainty due to measurement noise in the case where the data originates from sensors that are not collocated with the estimated forces. It is also investigated how the calculation time of the algorithm increases with the time delay assumed in the estimation. A steady-state initialization is proposed to enable a significant speed-up of the calculations. The presented algorithm is validated using data obtained from a field test on a footbridge. This allows to investigate the influence of modeling errors in the estimation, which are inevitable when dealing with real structures.

The outline of the paper is as follows. Section 2 presents the smoothing algorithm for joint input-state estimation and its application for response estimation. Next, Section 3 shows an illustration of the algorithm based on numerical simulations for a cantilever steel beam. Section 4 presents a validation of the smoothing algorithm using data obtained from a field test on a footbridge. Finally, in Section 5, the work is concluded.

2. Mathematical formulation

2.1. System model

Consider the following linear discrete-time combined deterministic-stochastic state-space description of a system:

$$\mathbf{x}_{[k+1]} = \mathbf{A}\mathbf{x}_{[k]} + \mathbf{B}\mathbf{p}_{[k]} + \mathbf{w}_{[k]} \tag{1}$$

$$\mathbf{d}_{[k]} = \mathbf{G}\mathbf{x}_{[k]} + \mathbf{J}\mathbf{p}_{[k]} + \mathbf{v}_{[k]} \tag{2}$$

where $\mathbf{x}_{[k]} \in \mathbb{R}^{n_\mathrm{s}}$ is the state vector, $\mathbf{d}_{[k]} \in \mathbb{R}^{n_\mathrm{d}}$ is the measured output vector, and $\mathbf{p}_{[k]} \in \mathbb{R}^{n_\mathrm{p}}$ is the input vector, to be estimated, with n_s the number of system states, n_d the number of outputs, and n_p the number of inputs. The system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{G}$, and $\mathbf{J}$ are assumed to be known. System noise is represented by the process noise vector $\mathbf{w}_{[k]} \in \mathbb{R}^{n_\mathrm{s}}$ and measurement noise vector $\mathbf{v}_{[k]} \in \mathbb{R}^{n_\mathrm{d}}$.

2.2. Smoothing algorithm

The system under consideration is described by Eqs. (1) and (2). Consider a vector $\mathbf{d}_{L[k]} \in \mathbb{R}^{(L+1)n_\mathrm{d}}$ that contains the response $\mathbf{d}_{[k]}$ over $L+1$ consecutive time steps ($L \geqslant 0$):

$$\mathbf{d}_{L[k]} \triangleq \begin{bmatrix} \mathbf{d}_{[k]} \\ \mathbf{d}_{[k+1]} \\ \vdots \\ \mathbf{d}_{[k+L]} \end{bmatrix} \tag{3}$$

Similar definitions are used for the input vector $\mathbf{p}_{L[k]}$ and the noise vectors $\mathbf{w}_{L[k]}$ and $\mathbf{v}_{L[k]}$. It is readily obtained from Eqs. (1)–(3) that:

$$\mathbf{d}_{L[k]} = \mathscr{O}_L\mathbf{x}_{[k]} + \mathscr{H}_L\mathbf{p}_{L[k]} + \mathscr{N}_L\mathbf{w}_{L-1[k]} + \mathbf{v}_{L[k]} \tag{4}$$