Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 292 (2015) 54–68
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

A priori and posteriori error analysis for time-dependent Maxwell's equations

Jichun Li[a,*], Yanping Lin[b]

[a] Department of Mathematical Sciences, University of Nevada Las Vegas, Las Vegas, Nevada 89154-4020, USA
[b] Department of Applied Mathematics, Hong Kong Polytechnic University, Hung Hom, Hong Kong

Available online 23 August 2014

Dedicated to Mary F. Wheeler on the occasion of her 75th birthday

Abstract

We consider time-dependent Maxwell's equations discretized by variable time steps in time domain and edge elements in spatial domain. First, the stability and optimal a priori error estimate are proved for both semi and fully discrete schemes. Then a posteriori error analysis is carried out for both schemes.

MSC: 78M10; 65N30; 35L15

Keywords: Maxwell's equations; Backward Euler scheme; Posteriori error analysis; Edge elements

1. Introduction

Wave propagation phenomena happen in a broad range of applications. Examples include sound waves, light waves and water waves, which arise in acoustics, electromagnetics, and fluid dynamics, respectively. The wave propagation problem is often described by the second-order hyperbolic equation, also called the wave equation. Upon considering time-harmonic (steady-state) waves, the wave equation reduces to the Helmholtz equation. Over the last four decades there has been considerable interest in developing various finite element methods (FEMs) for solving the wave equation (e.g., [1–6], and references therein). To solve the wave equation more efficiently, adaptive FEMs are often used. Adaptive FEMs are often based on a posteriori error estimates, i.e., some computable quantities that estimate the FEM solution error in a suitable norm.

Over the last three decades many a posteriori error estimates have been developed for time-independent problems such as elliptic equations and Helmholtz equation (e.g., [7–13], and references therein). As for time-dependent prob-

* Corresponding author. Tel.: +1 702 895 0365; fax: +1 702 895 4343.
E-mail addresses: jichun@unlv.nevada.edu (J. Li), malin@polyu.edu.hk (Y. Lin).

http://dx.doi.org/10.1016/j.cma.2014.08.009
0045-7825/

lems, considerable results on a posteriori error estimates for parabolic problems have also been obtained (e.g., [14–17], and references therein). However, according to Bernardi and Süli back in 2005 [18, p. 199]: "the a posteriori error analysis of finite element approximations to second-order hyperbolic problems is in a less complete state". Indeed, compared to results on elliptic, parabolic, and first order hyperbolic problems, "hyperbolic problems of second order have been much less studied" as Picasso mentioned in 2010 [19, p. 2213]. Though some a posteriori error estimates have been obtained over the years (e.g., [20–22,18,19]), "the design and implementation of adaptive algorithms for the wave equation based on rigorous a posteriori error estimators is a largely unexplored subject" as Georgoulis, Lakkis and Makridakis remarked in their 2013 paper [23, p. 1262].

Time-dependent Maxwell's equations are used in simulating electromagnetic wave propagation problems, and can be reduced to the second-order wave equation under special circumstances. Since the first a posteriori error estimate obtained for time-harmonic Maxwell's equations by Monk in 1998 [24], some excellent estimates have been proved (e.g., [25–29], [30, Ch. 6] and references therein). However, to the best of our knowledge, there seems no publication on a posteriori error estimates for time-dependent Maxwell's equations [31,32], except one paper by Zheng, Chen and Wang [33] for time-dependent eddy current problems and one very recent work by Creusé, Nicaise and Tittarelli [34] for the $A-\varphi$ magnetodynamic problem. Note that both problems of [33,34] are described by parabolic type equations, and many a posteriori error estimates techniques developed for parabolic equations can be applied. At the end of their 2005 paper [18], Bernardi and Süli mentioned that similar estimates for time-dependent Maxwell system can be shown using their techniques developed for the second-order wave equation, no such a study has ever been published so far. In this paper, we initiate this task by developing some a posteriori error estimates for time-dependent Maxwell's equations.

The outline of the paper is as follows. In Section 2, we present the model problem and a stability result. Following the technique introduced by Bernardi and Süli [18], Section 3 is devoted to the description of a backward Euler scheme with variable time steps, and a fully discrete edge element method. Stability and optimal a priori error estimate are proved for both the semi and fully discrete schemes. Section 4 is devoted to the a posteriori error analysis of both schemes. We conclude the paper in Section 5.

2. The governing equations and notation

Before we present the governing equations, let us introduce some notation. We assume that Ω is a bounded Lipschitz polyhedral domain in $\mathcal{R}^3$ with connected boundary $\partial\Omega$. For any domain $\omega \subset R^3$, we let $L^2(\omega)$ be the Hilbert space, equipped with inner product $(\cdot,\cdot)_{0,\omega}$ and norm $\|\cdot\|_{L^2(\omega)}$. When $\omega = \Omega$, we simply write $\|\cdot\|_{L^2(\Omega)} = \|\cdot\|$. We let $H^s(\omega)$ be the standard Sobolev space of order $s \geq 0$ equipped with norm $\|\cdot\|_{s,\omega}$ and semi-norm $|\cdot|_{s,\omega}$. For vector functions, we simply denote $\boldsymbol{L}^2(\omega) = (L^2(\omega))^3$ and $\boldsymbol{H}^s(\omega) = (H^s(\omega))^3$. To deal with Maxwell's equations, we need the spaces

$$H^s(\text{curl}; \Omega) = \left\{\boldsymbol{u} \in \boldsymbol{H}^s(\Omega);\ \nabla\times\boldsymbol{u} \in \boldsymbol{H}^s(\Omega)\right\}$$

and

$$H^s_0(\text{curl}; \Omega) = \left\{\boldsymbol{u} \in H^s(\text{curl}; \Omega);\ \hat{\boldsymbol{n}}\times\boldsymbol{u} = \boldsymbol{0} \text{ on } \partial\Omega\right\},$$

where $\hat{\boldsymbol{n}}$ is the unit outward normal to $\partial\Omega$. When $s = 1$, we simply write $H^1(\text{curl}; \Omega) = H(\text{curl}; \Omega)$ and $H^1_0(\text{curl}; \Omega) = H_0(\text{curl}; \Omega)$. For time $T > 0$ and any separable Banach space X, we consider the space $L^1(0, T; X)$ of integrable functions on $(0, T)$ with values in X. We also need the space $C^s(0, T; X)$ of continuously differentiable functions on $[0, T]$ up to the order s with values in X.

To model electromagnetic wave propagation, we usually have to solve the famous Maxwell's equations:

$$\epsilon\frac{\partial\boldsymbol{E}}{\partial t} - \nabla\times\boldsymbol{M} = -\boldsymbol{J}_s, \quad \text{in } \Omega\times(0,T), \tag{1}$$

$$\mu\frac{\partial\boldsymbol{M}}{\partial t} + \nabla\times\boldsymbol{E} = 0, \quad \text{in } \Omega\times(0,T), \tag{2}$$

where ϵ and μ denote the permittivity and permeability, respectively, $\boldsymbol{E}(\boldsymbol{x},t)$ and $\boldsymbol{M}(\boldsymbol{x},t)$ are the electric and magnetic fields, respectively, and $\boldsymbol{J}_s(\boldsymbol{x},t)$ is a given source. To make the problem well-posed, we further assume that the system