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Well-posedness and robust preconditioners for discretized fluid–structure interaction systems☆

Jinchao Xu, Kai Yang*

Department of Mathematics and Center for Computational Mathematics and Application, Pennsylvania State University, University Park, PA 16802, USA

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Highlights

- We formulate the discretized FSI system as saddle point problems.
- We analyze the well-posedness of the FSI saddle point problems.
- We propose optimal preconditioners for FSI discretized systems.

Abstract

In this paper we develop a family of preconditioners for the linear algebraic systems arising from the arbitrary Lagrangian–Eulerian discretization of some fluid–structure interaction models. After the time discretization, we formulate the fluid–structure interaction equations as saddle point problems and prove the uniform well-posedness. Then we discretize the space dimension by finite element methods and prove their uniform well-posedness by two different approaches under appropriate assumptions. The uniform well-posedness makes it possible to design robust preconditioners for the discretized fluid–structure interaction systems. Numerical examples are presented to show the robustness and efficiency of these preconditioners.

MSC: 74S05; 74F10; 65F08

Keywords: Fluid–structure interaction; Stabilization; Robust preconditioners

1. Introduction

Fluid–structure interaction (FSI) is a much studied topic aimed at understanding the interaction between some moving structure and fluid and how their interaction affects the interface between them. FSI has a wide range of applications in many areas including hemodynamics [1–4] and wind/hydro turbines [5–8].

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* Corresponding author.
E-mail address: yang_k@math.psu.edu (K. Yang).

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FSI problems are computationally challenging. The computational domain of FSI consists of fluid and structure subdomains. The position of the interface between fluid domain and structure domain is time dependent. Therefore, the shape of the fluid domain is one of the unknowns, increasing the nonlinearity of the FSI problems.

Many numerical approaches have been proposed to tackle the interface problem of FSI. The arbitrary Lagrangian–Eulerian (ALE) method is commonly used. ALE adapts the fluid mesh to match the displacement of structure on interface. Other approaches, such as the fictitious domain method [9,10] and the immersed boundary method [11–13], have inconsistent fluid and structure meshes and, therefore, need special treatment at the interface, such as interpolation between different meshes. In this paper, we focus on the ALE method.

There is much research focused on solving the fluid–structure interaction problem numerically using ALE formulation. These studies can be roughly classified into partitioned approaches and monolithic approaches [14]. Partitioned approaches employ single-physics solvers to solve the fluid and structure problems separately and then couple them by the interface conditions. Monolithic approaches solve the fluid and structure problems simultaneously. Depending on whether the interface conditions are exactly enforced at every time step, these approaches can also be classified into weakly and strongly coupled algorithms. Weakly coupled partitioned approaches are usually considered unstable due to the added-mass effect [15]. A semi-implicit approach proposed in [16] can avoid the added-mass effect for a wide range of applications, but it is subject to pressure boundary conditions. Several types of semi-implicit methods were proposed in [17,18]. Strongly coupled approaches are preferred for their stability. Although it is possible to achieve the strong coupling via partitioned solvers (by fixed-point iteration, for example), they usually introduce prohibitive computational costs due to slow convergence [19]. In this paper we consider monolithic approaches that strongly couple fluid variables with structure variables and we address some solver issues.

A great deal of work has been carried out to develop monolithic solvers for FSI [20–23]. In [24], a fully-coupled solution strategy is proposed to solve the FSI problem with large structure displacement. The nonlinearity is handled by Newton's method and various approaches to solve the Jacobian system are proposed. Block triangular preconditioners and pressure Schur complement preconditioners are used for the preconditioned Krylov subspace solvers. However, in [20] it is pointed out that block preconditioning for fluid and structure separately cannot resolve the coupling between fields and it is proposed that structure degrees of freedoms on interface be eliminated in order to effectively precondition degrees of freedom at the interface. In [23,25–27], a Newton–Krylov–Schwarz method for FSI is developed. Additive Schwarz preconditioners are used for Krylov subspace solvers and two-level methods are also developed. In [28,29], ILU preconditioners and inexact block-LU preconditioners are proposed to solve FSI problems.

In this paper, we reformulate semi-discretized systems of FSI as saddle point problems with fluid velocity, pressure and structure velocity as unknowns. The ALE mapping is decoupled from the solution of the velocity and pressure. Then, we carry out our theoretical analysis and solver design under this framework. With particular choice of norms, we prove that the saddle point problem is well-posed.

For the finite element discretization of FSI, we propose two approaches to prove the well-posedness. The first introduces a stabilization term to the fluid equations and the second adopts a norm of the velocity space that depends on the choice of the pressure space. Both of these approaches lead to uniform well-posedness of the finite element discretization of the FSI model under appropriate assumptions.

Based on the uniform well-posedness, we propose optimal preconditioners based on the framework in [30,31] such that the preconditioned linear systems have uniformly bounded condition numbers. Then, we compare the proposed preconditioners with the augmented Lagrangian preconditioners [32–35]. These preconditioners are all block preconditioners and their application requires efficient sub-block solvers. To test the preconditioners, we solve the linear systems coming from the discretization of the Turek and Hron benchmark problems [36]. The iteration counts of MINRES with several preconditioners are compared.

The rest of this paper is organized as follows. In Section 2, we introduce an FSI model and the ALE method. In Section 3, we study the proposed time and space discretization and its well-posedness. In Section 4, we propose optimal preconditioners for the discretized systems and demonstrate their performance with numerical examples.

2. An FSI model

We consider a domain $\Omega \subset \mathbb{R}^N (N = 2, 3)$ with a fluid occupying the upper half Ω_f and a solid occupying the lower half Ω_s, as illustrated in Fig. 1.