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Tooth-meshing-harmonic static-transmission-error amplitudes of helical gears

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ABSTRACT

The static transmission errors of meshing gear pairs arise from deviations of loaded tooth working surfaces from equispaced perfect involute surfaces. Such deviations consist of tooth-pair elastic deformations and geometric deviations (modifications) of tooth working surfaces. To a very good approximation, the static-transmission-error tooth-meshing-harmonic amplitudes of helical gears are herein expressed by superposition of Fourier transforms of the quantities: (1) the combination of tooth-pair elastic deformations and geometric tooth-pair modifications and (2) fractional mesh-stiffness fluctuations, each quantity (1) and (2) expressed as a function of involute "roll distance." Normalization of the total roll-distance single-tooth contact span to unity allows tooth-meshing-harmonic amplitudes to be computed for different *shapes* of the above-described quantities (1) and (2). Tooth-meshing harmonics $p = 1, 2, \ldots$ are shown to occur at Fourier-transform harmonic values of Qp, $p = 1, 2, \ldots$, where Q is the actual (total) contact ratio, thereby verifying its importance in minimizing transmission-error tooth-meshing-harmonic amplitudes. Two individual shapes and two series of shapes of the quantities (1) and (2) are chosen to illustrate a wide variety of shapes. In most cases representative of helical gears, tooth-meshing-harmonic values $p = 1, 2, \ldots$ are shown to occur in Fourier-transform harmonic regions governed by discontinuities arising from tooth-pair-contact initiation and termination, thereby showing the importance of minimizing such discontinuities. Plots and analytical expressions for all such Fourier transforms are presented, thereby illustrating the effects of various types of tooth-working-surface modifications and tooth-pair stiffnesses on transmission-error generation.

1. Introduction

The static transmission error (STE) is widely recognized to be the dominant source of vibration excitation caused by meshing gear pairs [1–13]. A fictitious pair of meshing gears with equispaced rigid perfect involute tooth-working-surfaces would yield zero transmission error. But under loading, the teeth of real gears elastically deform [14–20]; moreover, real gear teeth have spacing and other manufacturing errors [21–23]. Therefore, in order to avoid impact loading and transmission-error step (jump) discontinuities, tooth working surfaces entering and leaving the mesh generally are modified by removal of material from otherwise perfect involute surfaces. In the case of helical gears, such modifications take the form of some sort of "end relief," "crowning," "generated engagement relief," or "bias" modifications, etc. [24–39]. All such

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modifications and tooth/gearbody elastic deformations generate STE contributions which can serve as excitations in gearing dynamics studies [12,40–42]. Moreover, tooth sliding friction is an additional source of vibration excitation [43–45].

Transmission-error analyses often are carried out in the frequency domain. For accurately manufactured gears, the dominant STE harmonic contributions are the tooth-meshing harmonics, which are caused by tooth/gearbody elastic deformations and geometric deviations from perfect involute surfaces of the averaged working surfaces of the teeth on each of the meshing gears [46, p. 112]. A formulation is provided herein to enable further understanding of the roles of tooth elastic deformations, tooth-working-surface modifications, and mesh stiffness fluctuations on STE tooth-meshing-harmonic contributions of helical gears.

In the following pages it is shown that the STE tooth-meshing-harmonic contributions, from a meshing helical gear pair, can be decomposed into two components: (1) the combination of tooth-pair elastic deformations and stiffness-weighted geometric tooth-pair modifications and (2) individual tooth-pair stiffnesses. The Fourier series amplitudes of the STE tooth-meshing harmonics then are shown to be determined by differences of the Fourier transforms of the above-mentioned two components. Normalization of the total contact span of individual mating tooth pairs to unity allows different *shapes* of the above-mentioned components (1) and (2) to be studied, illustrating their STE contributions for any (total) contact ratio. A significant motivation for the study is determining how discontinuities in the above two components (1) and (2), encountered at tooth-pair contact initiation and termination, affect transmission error amplitudes. Two individual models and two series of models, applicable to either of the above-mentioned two components, illustrate a range of discontinuities to accomplish this goal. In particular, it is suggested how different types of tooth-pair modifications are likely to affect such discontinuities, including possible approaches to minimize their transmission error contributions. Apart from the *insight* provided by these examples, the simplicity of the final analytical formulation, provided in Section 8, should be useful in other studies and in understanding experimental results.

Following [4,5], an "exact" derivation of static-transmission-error contributions is provided in [46], which is the starting point for the work. It is shown that the combined transmission-error contribution from tooth-pair elastic deformations and working-surface deviations can be expressed as a "rep function" [47, p. 28,46, p. 229] of the individual tooth-pair contributions, thereby allowing the transmission-error Fourier series amplitudes of this combined tooth-pair elastic-deformation/working-surface-deviation contribution to be expressed by the Fourier transform of this combined contribution evaluated at "frequency" locations Qp, where Q is the actual (total) contact ratio, and p = 1, 2, ... are the transmission-error tooth-meshing harmonics, thereby providing a direct relationship between this *source* of the transmission-error tooth-meshing harmonics and the tooth-meshing-harmonic amplitudes. After normalizing tooth-pair roll-distance spans to unity, two different *shapes* and two series of *shapes* of this combined contribution are delineated, and their Fourier transforms computed, thereby showing the relationships between the tooth-working-surface sources of the transmission error and transmission-error tooth-meshing-harmonic amplitude contributions from this source. In most of these examples, tooth-meshing-harmonic amplitudes are shown to fall in the harmonic regions controlled by tooth-pair-contact initiation and termination, thereby illustrating the dominant importance of discontinuities at tooth contact initiation and termination, and of *actual* (total) contact ratios.

Following the above-described treatment, it is shown that the mesh-stiffness also can be expressed as a "rep function" of individual tooth-pair stiffnesses, thereby allowing the fractional mesh-stiffness-fluctuation transmission-error Fourier series contribution to be expressed by the Fourier transform of the stiffness of individual mating tooth pairs evaluated at "frequency" locations Qp, where Q is the actual (total) contact ratio, and p = 1, 2, ... are transmission-error tooth-meshing harmonics, as above. Each of the two series of shapes mentioned above also can be suitable for representing the tooth-pair stiffness contributions.

The final transmission-error Fourier series representation then is obtained by combining the contributions from the two above-described sources. The resultant model is representative of the physical generation of transmission-error tooth-meshing-harmonic contributions of helical gears.

2. Lineal transmission error of parallel-axis gear pairs

Fig. 1 illustrates a meshing pair of parallel-axis helical or spur gears with rigid equispaced perfect involute teeth. Such a gear pair would transmit an exactly constant speed ratio. A single independent variable x, roll distance [8,46], can be used to designate the rotational position of the two meshing gears,

$$x \triangleq R_b^{(1)}\theta^{(1)} = R_b^{(2)}\theta^{(2)}, \tag{1}$$

where $R_b^{(1)}$ and $R_b^{(2)}$ denote the base-cylinder radii of the two perfect gears (1) and (2), and $\theta^{(1)}$ and $\theta^{(2)}$ denote their instantaneous rotational positions. Let $\delta\theta^{(1)}(x)$ and $\delta\theta^{(2)}(x)$ denote the instantaneous rotational deviations of gears (1) and (2), respectively, from the rotational positions $\theta^{(1)}$ and $\theta^{(2)}$ of their perfect involute counterparts. Assume the gear shafts remain parallel and fixed. Define the lineal transmission error, $\zeta(x)$, as the amount the teeth come together in the plane of contact, as in elastic deformations, relative to their perfect involute counterparts. Then [8,46], the lineal transmission error $\zeta(x)$ is

$$\zeta(x) \triangleq R_b^{(1)}\delta\theta^{(1)}(x) - R_b^{(2)}\delta\theta^{(2)}(x), \tag{2}$$

where the negative sign in Eq. (2) arises from the sign convention of $\theta^{(2)}$ illustrated in Fig. 1.