Mechanical Systems and Signal Processing 98 (2018) 652–666

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Bayesian operational modal analysis with asynchronous data, part I: Most probable value

Yi-Chen Zhu *, Siu-Kui Au

Institute for Risk and Uncertainty and Centre for Engineering Dynamics, University of Liverpool, United Kingdom

ARTICLE INFO

Article history:
Received 26 October 2016
Received in revised form 2 April 2017
Accepted 19 May 2017
Available online 29 May 2017

Keywords:
Ambient data
Asynchronous data
Bayesian methods
FFT
Operational modal analysis

ABSTRACT

In vibration tests, multiple sensors are used to obtain detailed mode shape information about the tested structure. Time synchronisation among data channels is required in conventional modal identification approaches. Modal identification can be more flexibly conducted if this is not required. Motivated by the potential gain in feasibility and economy, this work proposes a Bayesian frequency domain method for modal identification using asynchronous 'output-only' ambient data, i.e. 'operational modal analysis'. It provides a rigorous means for identifying the global mode shape taking into account the quality of the measured data and their asynchronous nature. This paper (Part I) proposes an efficient algorithm for determining the most probable values of modal properties. The method is validated using synthetic and laboratory data. The companion paper (Part II) investigates identification uncertainty and challenges in applications to field vibration data.

1. Introduction

Modal identification aims at determining the modal characteristics of a structure, which primarily include natural frequencies, damping ratios and mode shapes [1,2]. These properties are demanded for industrial applications such as vibration control, damage detection and structural health monitoring [3–6]. Compared to traditional vibration tests based on free vibration (zero input) or forced vibration (known input), ambient vibration test does not require artificial loading but assumes that it is broadband random. Based on random natural excitations such as wind, microtremor and cultural activities, ambient vibration test can be conducted during the daily use of structure. For its high economy and convenience in applications, it has attracted much interest in both theory development and field test applications over the past few decades [7–9].

Modal identification based on 'output-only' ambient vibration data is conventionally known as 'operational modal analysis' (OMA) [10,11]. Among other methods, frequency domain decomposition [12] provides a quick estimation based on sample power spectral density (PSD) [13]. Stochastic subspace identification [14–16] extracts modal parameters by least square estimation with a state-space model. Recently, transmissibility based OMA techniques have been proposed [17,18] with the premise of being robust to the characteristics of the excitation spectra in identifying mode shapes.

Bayesian approach views modal identification as a general inference problem based on available information. Methods [19,20] have been developed in different contexts, in the time domain [21], frequency domain based on sample PSD matrix [22–24] and Fast Fourier transform (FFT) of data [25,26]. Given information of data and modelling assumptions, the infor-

* Corresponding author at: Harrison Hughes Building, Brownlow Hill, Liverpool L69 3GH, United Kingdom.
E-mail addresses: sgyzhu7@liverpool.ac.uk (Y.-C. Zhu), siukuiau@liverpool.ac.uk (S.-K. Au).

http://dx.doi.org/10.1016/j.ymssp.2017.05.027
0888-3270/

mation about modal parameters is extracted using Bayes' theorem and encapsulated in the 'posterior' (i.e. given data) probability density function (PDF). For modal identification problem which is 'globally identifiable' [27], the PDF can be characterised by the most probable value (MPV, where it is peaked) and covariance matrix (reflecting identification uncertainty).

In order to obtain mode shape information, the vibrations at different locations of a structure are measured using multiple sensors. Conventional modal identification techniques require 'synchronous' data, where the digital data at different channels are sampled according to the same time scale. Synchronisation does not only mean that the data recorded at multiple channels should start at the same time but also at the same pace. Simply logging multiple channels of data on the same computer does not imply they are synchronised. Each data acquisition (DAQ) unit has its own clock for sampling, whose accuracy is affected by temperature, aging etc. [28]. When multiple data channels are recorded using independent DAQ units, they will not be perfectly synchronised.

Practically, synchronisation means that the sampling time difference between data channels is within a certain tolerance. In one conventional configuration, the analog signals of sensors are sampled using a central synchronisation hardware. In full-scale tests, this requires long analog cables, which inevitably introduce additional noise in the data. Alternative options are Network Time Protocol [29] through the internet or Global Positioning System [30] for outdoor applications. Wireless sensor networks have also been applied in vibration tests with synchronisation corrections [31–33].

Synchronisation comes with an overhead. The possibility of performing OMA with asynchronous data is worth exploring. Motivated by the above considerations, a Bayesian method is proposed in this work for modal identification using asynchronous output-only ambient data. Asynchronous data is generally a non-stationary process, which is difficult to model from first principles. A stationary model with imperfect coherence is proposed so that it is conducive to modal identification, while capturing the key asynchronous characteristics within suitable time scales. Based on this model, the likelihood function for Bayesian inference is derived and its mathematical structure is analysed. An algorithm is developed in this paper (Part I) for efficiently determining the MPV of modal parameters. The companion paper (Part II) focuses on efficient determination and investigation of identification uncertainty. Synthetic and laboratory data examples are presented to illustrate and verify the proposed method. An application to modal identification of a full-scale building is also presented.

This paper is organised as follow. Bayesian approach based on FFT of data in a general context is briefly reviewed in Section 2. An identification model for asynchronous data is presented in Section 3. Based on this model, the mathematical structure of the theoretical PSD matrix for asynchronous data is analysed in Section 4. To facilitate analysis and modal identification, simplifying assumptions are made and the resulting posterior PDF is derived in Section 5. An iterative algorithm for determining the MPV of modal parameters is developed in Section 6. The high signal-to-noise ratio asymptotic behaviour of the MPV is analysed in Section 7. The overall identification procedure is summarised in Section 8. The proposed algorithm for MPV is validated using synthetic and laboratory data in Section 9.

2. Bayesian framework

The proposed modal identification method is based on Bayesian approach using the FFT of ambient data for probabilistic inference. The overall framework is briefly reviewed in this section. Input loading is unknown but assumed to be broadband random near the resonance band of the modes of interest. Let $\{\hat{\ddot{\mathbf{x}}}_j \in R^n\}_{j=1}^N$ denote the measured ambient acceleration data at n degrees of freedom (DOF) of the subject structure; N is the number of samples per channel. It is assumed to consist of theoretical structural response $\ddot{\mathbf{x}}_j \in R^n$ under ambient excitation and prediction error $\boldsymbol{\varepsilon}_j \in R^n$:

$$\hat{\ddot{\mathbf{x}}}_j = \ddot{\mathbf{x}}_j + \boldsymbol{\varepsilon}_j \quad (1)$$

The prediction error accounts for the difference between the theoretical response and measured data, which may arise from measurement noise or modelling error. The 'scaled FFT', or FFT in short, of $\{\hat{\ddot{\mathbf{x}}}_j\}$ is defined as:

$$\mathsf{F}_k = \sqrt{\frac{2\Delta t}{N}} \sum_{j=1}^{N} \hat{\ddot{\mathbf{x}}}_j \exp\left[-2\pi\mathbf{i}\frac{(k-1)(j-1)}{N}\right] \quad (2)$$

where $\mathbf{i}^2 = -1$ and Δt is the sampling interval. Here, F_k corresponds to frequency $\mathsf{f}_k = (k-1)/N\Delta t$ (Hz) for $k = 1,\ldots,\mathrm{N_q}$, where $\mathrm{N_q}$ (integer part of N/2 + 1) is the index corresponding to the Nyquist frequency. Multiplying F_k by its conjugate transpose gives the sample PSD matrix. The scaling factor is defined such that the PSD is one-sided with respect to frequency in Hz. For modal identification, only the F_k within a selected frequency band dominated by the modes of interested is used.

Let $\boldsymbol{\theta}$ denote the set of modal parameters to be identified. Using Bayes' theorem with a uniform prior PDF, the posterior PDF of $\boldsymbol{\theta}$ given $\{\mathsf{F}_k\}$ is

$$p(\boldsymbol{\theta}|\{\mathsf{F}_k\}) \propto p(\{\mathsf{F}_k\}|\boldsymbol{\theta}) \quad (3)$$

where $p(\{\mathsf{F}_k\}|\boldsymbol{\theta})$ is called the 'likelihood function'. A uniform prior PDF is justified for modal identification because the typical data size is sufficiently large that the likelihood function is fast-varying compared to the pior PDF. Assuming that data is stochastic stationary, for long data duration and high sampling rate (i.e. large $N\Delta t$ and small Δt), it can be shown that