Mechanical Systems and Signal Processing 98 (2018) 834–851

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Feasibility of motion laws for planar one degree of freedom linkage mechanisms at dead point configurations

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ARTICLE INFO

Article history:
Received 2 September 2016
Received in revised form 5 May 2017
Accepted 13 May 2017
Available online 3 June 2017

Keywords:
Inverse kinematics
Linkage mechanism
Singular configuration
Dead point
Motion law

ABSTRACT

This paper proposes an analytical solution of the Inverse Kinematics (IK) problem at dead point configurations for any planar one degree of freedom linkage mechanism, with regard to the continuity $\mathcal{C}^n$ of the motion law. The systems analyzed are those whose elements are linked with lower pairs and do not present redundancies. The study aims to provide the user with some rules to facilitate the design of feasible motion profiles to be reproduced by conventional electrical actuators at these configurations. During the last decades, several methods and techniques have been developed to study this specific configuration. However, these techniques are mainly focused on solving numerically the IK indeterminacy, rather than analyzing the motion laws that the mechanisms are able to perform at these particular configurations. The analysis presented in this paper has been carried out differentiating and applying l'Hôpital's rule to the system of constraint equations $\boldsymbol{\phi}(\boldsymbol{q})$ of the mechanism. The study also considers the feasibility of the time-domain profiles to be reproduced with conventional electrical actuators (i.e. AC/DC motors, linear actuators, etc.). To show the usefulness and effectiveness of the method, the development includes the analytical application and numerical simulations for two common one degree of freedom systems: a slider-crank and a four linkage mechanisms. Finally, experimental results are presented on a four linkage mechanism test bed.

1. Introduction

The design of motion profiles of machine elements has been widely studied in research areas such as mechanical engineering and control engineering. Commonly, the transmission chain between the actuator coordinate, commanded by the user, and the coordinate that describes the desired motion, is linear (pulley belt [1], gear chain [2], rack and pinion, etc.) (Fig. 1). In this case, the constraints and limitations to perform any profile generally come from the actuator or control performances. Nevertheless, when the transmission chain is a linkage mechanism, it is necessary to deal with additional constraints, such as the singularities of the targeted coordinate within its functional range [3]. The actuator coordinate, however, has typically no singularities.

In the study of kinematics in linkage mechanisms with planar motion, the mobility can be described by a set of generalized coordinates and generalized velocities. The coordinates to describe the configuration of the mechanism are named generalized coordinates. The minimum number of them to fully define the state of the mechanism are the so-called independent

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http://dx.doi.org/10.1016/j.ymssp.2017.05.015

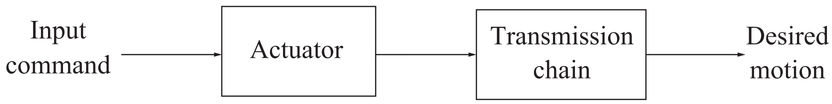

Fig. 1. Block diagram of the system.

coordinates, while the rest are known as dependent coordinates. In the case of a planar one degree of freedom mechanism, there is only one independent coordinate. Generically, the aim of the user is to design a time domain evolution of this coordinate, described by means of a motion law or profile. Thus, the inverse kinematics (IK) approach could be used to calculate the actuator command. However, if this coordinate has dead point configurations within its accessible configuration range, IK leads to algebraic indeterminacies. This paper addresses which type of time-domain profiles can be performed, by the independent coordinate, in presence of dead points. The analysis is carried out with regard to the continuity $\mathcal{C}^n$ of the profile and its feasibility to be reproduced with conventional electrical actuators (i.e. AC/DC motors, linear actuators, etc.).

In order to find a solution to IK near dead points, Whitney [4] suggested using the pseudoinverse of the Jacobian matrix (J). The main drawback of this method is that it has stability problems in the neighborhood of dead points. Balestrino et al. [5] and Wolovich and Elliot [6] proposed to use the transpose of J instead of its inverse which, according to Buss [7], is a good approximation when scaled by some small scalar α. Another approach is the well-known *damped least-squares* method (DLS) introduced by Nakamura and Hanafusa [8] and Wampler [9]. This technique uses a damping factor k to ensure that an inverse kinematic solution exists in the vicinity of a dead point by allowing the independent coordinate to deviate from the reference trajectory. There have been several authors that presented methods to find an appropriate value for k ([8,10–15]). Similar methods to the DLS are exposed by Xiang et al. [16] and Sugihara [17].

Another approach to solve the IK problem is based on a null-space path tracking technique presented by Nenchev [18], and Nenchev and Uchiyama [19]. This method, based on the work of Kieffer [20] and known as *Singularity Consistent* (SC) method, proposes to reparametrize the path of the independent coordinate in the neighborhood of a dead point. The main idea is to parametrize the independent coordinate trajectory with a smooth function $g(\alpha)$, where α is not time and is treated as a dependent coordinate. Nenchev et al. [21] analytically demonstrated the equivalence between the *Singularity Consistent* and the *Jacobian adjoint matrix* technique used by Tchoń and Dulęba [22] and Senft and Hirzinger [23].

Other methods for solving the IK problem are based on artificial neural networks ([24,25]). Aristidou and Lasenby [26] suggested a heuristic method called *Forward And Backward Reaching Inverse Kinematics* (FABRIK) that takes the last calculated position of the dependent coordinates to find the future values in a forward and backward iterative mode. The *Feedback Inverse Kinematics* (FIK) presented by Pechev [27] uses a feedback loop to minimize the difference between the actual and the target velocity. Vargas [28] introduced the *Filtered Inverse* algorithm (FI), which dynamically estimates the inverse of the Jacobian matrix J.

The present paper proposes an analytical solution of the IK problem at dead point configurations, for any planar one degree of freedom linkage mechanism, with regard to the continuity $\mathcal{C}^n$ of a motion law that describes the time-evolution of the independent coordinate. The systems analyzed are those whose elements are linked with lower pairs and do not present redundancies. The novelty of this study is to provide the user with some rules to facilitate the design of feasible motion profiles of the independent coordinate at these configurations, while dealing with the limitations of conventional electrical actuators. The main contributions of this development are: (*i*) The study of the feasibility of the time-domain profiles, with regard to the continuity $\mathcal{C}^n$, to be reproduced with the above-mentioned actuators; (*ii*) The analysis carried out by differentiating and applying l'Hôpital's rule to the system of constraint equations $\boldsymbol{\phi}(\boldsymbol{q})$ of the mechanism, which is a different approach from previous work; (*iii*) The specific rules to design motion profiles around a dead point configuration, taking into consideration the limitations of these actuators and all the casuistry involved, related with the values of the derivatives of the independent coordinate, and (*iv*) the verification of the conclusions obtained through simulation and experimental results.

The paper is organized as follows. Section 2 introduces the problem formulation of the IK approach in the neighborhood of a dead point. Section 3 analyzes the limitations of a conventional electrical actuator to perform a command input with regard to the continuity $\mathcal{C}^n$. Section 4 proposes the workspace solutions of the problem stated in Section 2 and particularizes the results for a slider-crank mechanism. Some simulations to confirm the goodness of the development are carried out in Section 5 and experimental results are presented in Section 6. Finally, conclusions are drawn in Section 7.

2. Problem formulation

The mobility of a mechanism of m degrees of freedom can be described by a set of $n+m$ generalized coordinates $\boldsymbol{q} = \{q_1, q_2, \ldots, q_n, q_{n+1}, \ldots, q_{n+m}\}^{\mathrm{T}}$ and generalized velocities $\dot{\boldsymbol{q}} = \{\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n, \dot{q}_{n+1}, \ldots, \dot{q}_{n+m}\}^{\mathrm{T}}$. The set of geometric variables $\boldsymbol{q}$ defines all the possible configurations of the mechanism, while its derivatives $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ define the distribution of the mechanism velocities and accelerations. The minimum set of m coordinates to fully define the configuration of the mechanism are called independent coordinates ($\boldsymbol{q}^{\mathrm{i}} = \{q_{n+1}, \ldots, q_{n+m}\}^{\mathrm{T}}$), while the remaining n are known as dependent coordinates ($\boldsymbol{q}^{\mathrm{d}} = \{q_1, q_2, \ldots, q_n\}^{\mathrm{T}}$). In the case of a planar one degree of freedom mechanism ($m = 1$), there is only one