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Bayesian operational modal analysis with asynchronous data, Part II: Posterior uncertainty

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ABSTRACT

A Bayesian modal identification method has been proposed in the companion paper that allows the most probable values of modal parameters to be determined using asynchronous ambient vibration data. This paper investigates the identification uncertainty of modal parameters in terms of their posterior covariance matrix. Computational issues are addressed. Analytical expressions are derived to allow the posterior covariance matrix to be evaluated accurately and efficiently. Synthetic, laboratory and field data examples are presented to verify the consistency, investigate potential modelling error and demonstrate practical applications.

1. Introduction

The identification uncertainty of modal parameters (e.g. natural frequencies, damping ratios and mode shapes) provides important information for risk assessment and structural health monitoring [1,2]. In operational modal analysis (OMA), the loading information is unknown and its intensity and frequency characteristics cannot be directly controlled. The identification uncertainty is often significantly larger than those in known input modal tests (like forced vibration or free vibration tests). Quantifying and Managing the uncertainty of identified modal parameters then becomes important for OMA.

For non-Bayesian or 'frequentist' methods, identification uncertainty is often assessed in terms of the ensemble variance of estimates over repeated experiments. Some challenges are discussed in [3]. For stochastic subspace identification (SSI), computational methods have been developed based on first-order perturbation for single setup data [4,5] and multi-setup data [6]. See also [7] for the variance of maximum likelihood modal parameter estimator in the state-space time domain. In a Bayesian context [8], identification uncertainty is quantified in terms of the covariance matrix associated with the 'posterior' (i.e. given data) distribution of modal parameters. For globally identifiable problems where the distribution has a single peak, the 'posterior covariance matrix' can be approximated by the inverse of Hessian of the negative log-likelihood function (NLLF) [9]. For OMA with synchronous data, efficient methods have been developed in different settings, e.g., well-separated modes [10], close modes [11] and multiple setups [12]. Mathematical connection between Bayesian and frequentist quantification of identification uncertainty has also been discussed [13]. Analytical expressions for the posterior covariance matrix have been derived under asymptotic conditions of long data and small damping, revealing the achievable

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identification precision of OMA [14]. See also [15] for work on related objectives but approached from a frequentist perspective for free vibration data.

A fast Bayesian OMA formulation for asynchronous data has been proposed in the companion paper; and an efficient method for determining the most probable values (MPV) of modal parameters has been developed. This paper investigates the posterior uncertainty of modal parameters and its efficient computation. Synthetic and laboratory data examples are presented to illustrate and verify the proposed OMA method. A field data example is also presented to illustrate real applications.

This paper is organized as follow. In Section 2, the NLLF for asynchronous data developed in the companion paper is briefly reviewed. In Section 3, computational issues associated with the posterior covariance matrix are discussed. Analytical expressions for the Hessian of NLLF (whose inverse gives the covariance matrix) are derived to allow accurate and efficient computation. The procedure for calculating the posterior covariance matrix is also summarised. In Section 4, synthetic, laboratory and field test examples are presented to illustrate the proposed method. Computational time is addressed in Section 5. Some comments regarding the practical issues are discussed in Section 6. The paper is concluded in Section 7.

2. NLLF for asynchronous OMA data

The posterior covariance matrix of modal parameters can be obtained as the inverse of the Hessian of negative log-likelihood function (NLLF). Consider the case of a well-separate mode where only one mode is dominant in the selected frequency band. Assume zero coherence among data of different synchronous data groups, it is shown in the companion paper that the NLLF is given by

$$L = \sum_{i=1}^{n_g} L_i \tag{1}$$

where

$$L_i = (n_i - 1)N_f \ln S_{ei} + \sum_k \ln(SD_k c_i + S_{ei}) + S_{ei}^{-1}(d_i - \bar{\mathbf{u}}_i^T \mathbf{A}_i \bar{\mathbf{u}}_i) \tag{2}$$

$$D_k = \left[(\beta_k^2 - 1)^2 + (2\zeta\beta_k)^2\right]^{-1} \quad \beta_k = f/\mathrm{f}_k \tag{3}$$

$$d_i = \sum_k \mathrm{F}_{ik}^* \mathrm{F}_{ik} \tag{4}$$

$$\mathbf{A}_i = \sum_k (1 + S_{ei}/SD_k c_i)^{-1} \mathbf{D}_{ik} \tag{5}$$

$$\mathbf{D}_{ik} = \mathrm{F}_{ik} \mathrm{F}_{ik}^* \tag{6}$$

In the above equations, F_{ik} is the FFT of measured data associated with the ith synchronous group corresponding to frequency f_k in the selected frequency band; N_f is the number of FFT data in the band; f and ζ denote the natural frequency and damping ratio of the mode, respectively; $\mathbf{u}_i \in R^{n_i}$ is the mode shape measured by the ith group with n_i degrees of freedom (DOF); n_g is the total number of synchronous data groups; $c_i = \|\mathbf{u}_i\|^2$ and $\bar{\mathbf{u}}_i = \mathbf{u}_i/\|\mathbf{u}_i\|$ so that $\|\bar{\mathbf{u}}_i\| = 1$; S is the modal force PSD (power spectral density) and S_{ei} is the prediction error PSD of the ith group.

3. Posterior uncertainty

The Hessian matrix of NLLF is a symmetric matrix containing the second derivatives of L with respect to (w.r.t.) $\boldsymbol{\theta} = \{f, \zeta, S, \{S_{ei}\}_{i=1}^{n_g}, \boldsymbol{\varphi}\}$. These derivatives will be derived analytically in this section, allowing an accurate and efficient determination of Hessian without resorting to finite difference method.

The function L_i in Eq. (2) is first written explicitly in terms of the global mode shape $\boldsymbol{\varphi}$ to facilitate differentiation. Let $\mathbf{L}_i \in R^{n_i \times n}$ be a selection matrix so that $\mathbf{L}_i\boldsymbol{\varphi}$ gives the local mode shape confined to the DOFs in the ith group. The (j,k)-entry of $\mathbf{L}_i$ is equal to 1 if DOF k is measured by the jth channel in the ith synchronous group, and zero otherwise. Then c_i and $\bar{\mathbf{u}}_i$ can be expressed in terms of $\boldsymbol{\varphi}$:

$$c_i = \|\mathbf{L}_i\boldsymbol{\varphi}\|^2 = \boldsymbol{\varphi}^T \mathbf{L}_i^T \mathbf{L}_i \boldsymbol{\varphi} \tag{7}$$

$$\bar{\mathbf{u}}_i = \frac{\mathbf{L}_i\boldsymbol{\varphi}}{\|\mathbf{L}_i\boldsymbol{\varphi}\|} = (\boldsymbol{\varphi}^T \mathbf{L}_i^T \mathbf{L}_i \boldsymbol{\varphi})^{-1/2} \mathbf{L}_i \boldsymbol{\varphi} \tag{8}$$

The global mode shape is subjected to unit norm constraint, i.e.,