Mechanical Systems and Signal Processing 98 (2018) 1000–1019

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Circularly-symmetric complex normal ratio distribution for scalar transmissibility functions. Part III: Application to statistical modal analysis

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ARTICLE INFO

Article history:
Received 26 December 2016
Received in revised form 16 May 2017
Accepted 22 May 2017
Available online 9 June 2017

Keywords:
Transmissibility
Probabilistic model
Modal analysis
Harmonic component
Uncertainty quantification

ABSTRACT

This study applies the theoretical findings of circularly-symmetric complex normal ratio distribution Yan and Ren (2016) [1,2] to transmissibility-based modal analysis from a statistical viewpoint. A probabilistic model of transmissibility function in the vicinity of the resonant frequency is formulated in modal domain, while some insightful comments are offered. It theoretically reveals that the statistics of transmissibility function around the resonant frequency is solely dependent on 'noise-to-signal' ratio and mode shapes. As a sequel to the development of the probabilistic model of transmissibility function in modal domain, this study poses the process of modal identification in the context of Bayesian framework by borrowing a novel paradigm. Implementation issues unique to the proposed approach are resolved by Lagrange multiplier approach. Also, this study explores the possibility of applying Bayesian analysis in distinguishing harmonic components and structural ones. The approaches are verified through simulated data and experimentally testing data. The uncertainty behavior due to variation of different factors is also discussed in detail.

1. Introduction

Structural health monitoring (SHM) has been an emerging field due to its ability of assessing the health condition of a structure [3–5]. Signal processing such as fast Fourier transform (FFT) is of fundamental importance in the context of SHM. Transmissibility function, which is widely used to characterize the dynamics of a system as a mathematical representation of the output-to-output relationship in the frequency domain [6], has been regarded to be a good candidate for SHM by different research groups [7–19].

As an alternative to the conventional output-only modal identification approaches in the frequency domain, transmissibility-based operational modal analysis (TOMA) originally proposed by Devriendt and Guillaume [8,9] has aroused considerable interest over the past few years. They revealed that transmissibility functions converge to an amplitude ratio of vibration modes while approaching the system poles, indicating that modal parameters can be estimated by combining transmissibility functions under different loading conditions. TOMA was compared with power spectrum-based modal analysis approach in [10] to illustrate its capacity of reducing the risk of wrongly identifying the modal parameters.

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http://dx.doi.org/10.1016/j.ymssp.2017.05.029

TOMA appears very appealing amongst recently suggested methods in that it is insensitive to the excitation characteristics. Following the work of [8,9], several new strategies on how to identify modal parameters from scalar transmissibility functions have been proposed more recently [11–16]. The approach to identify modal parameters from scalar transmissibility functions was also generalized to transmissibility matrix (multivariate transmissibility) [17,18].

The theories of deterministic analysis using transmissibility measurements are relatively mature. In reality, the inherent randomness of measurement and variability of environmental condition [19] inevitably introduce variability leading to misinterpreted identification results. Rare TOMA approaches aforementioned can provide a rigorous quantification of the uncertainties of the modal parameters, which is important in modern SHM. In this regard, the issue of characterization of uncertainty in TOMA is worth of further investigation.

This study will pose the process of transmissibility-based modal analysis in the context of Bayesian framework attributed to its ability in resolving uncertainties in the context of probability logic [20–31]. Following the statistics of measurements in the frequency or time domain, a number of methods [32–38] have been proposed. Given measured data and modeling assumptions, these methods provide rigorous means for obtaining modal properties as well as their uncertainties. To address the computational challenges of the conventional Bayesian FFT approach, a significant breakthrough contribution was made recently [39–44]. As a new paradigm, fast Bayesian operational modal analysis appears appealing due to its rigorous derivation, high efficiency and easy-implemented features [39]. The ideas and techniques have been borrowed in different manners [45,46].

It has been proved by the authors that raw transmissibility functions follow circularly-symmetric complex normal ratio distribution [1,2]. As a sequel to the development, a probabilistic model connecting modal parameters and transmissibility measurements in the vicinity of the resonant frequency is developed in this study. One can figure out that, when approaching the stystem poles, the statistics of transmissibility function is dependent on 'noise-to-signal' ratio and mode shapes. On the basis of the probability density function (PDF) of raw transmissibility function, the modal identification problem is formulated as one minimizing a negative log-likelihood function (NLLF) as an objective function in the context of the novel fast Bayesian operational modal analysis paradigm [39]. A Lagrange multiplier method is required to compute the most probable values rapidly, while the covariance matrix is obtained by taking the inverse of Hessian matrix which can be derived analytically. Furthermore, this study investigates the possibility of using Bayesian analysis in distinguishing harmonic components and structural components. Experimental studies are employed to verify the accuracy of the proposed method and the variation of uncertainty with respect to different factors. The research paradigm has the potential to be further extended to other applications such as model updating, damage detection and transfer path analysis, etc. by fitting a parametric model of transmissibility function in the Bayesian framework.

2. Probabilistic model of raw scalar transmissibility function in modal domain

2.1. Scalar transmissibility function

Consider a linear system with n_d dofs subjected to a stationary excitation, and assume that discrete acceleration responses are available for n_o ($\leqslant n_d$) measured dofs. The sampling time interval is assumed to be Δt and the time duration is assumed to be T_d. The Fast Fourier transform (FFT) of $\mathbf{y}(n)$ at frequency f_k is defined as

$$\mathbf{Y}_k = \mathbf{Y}_k^{\Re} + \mathbf{i}\mathbf{Y}_k^{\Im} = \sqrt{\frac{\Delta t}{2\pi N}}\sum_{n=0}^{N-1}\mathbf{y}(n)\exp(-\mathbf{i}2\pi f_k n\Delta t) \tag{1}$$

where $n = 1, 2, \cdots, N$, $\mathbf{i}^2 = -1$, $f_k = k\Delta f$, $k = 1, 2, \cdots, Int(N/2)$. For raw FFT coefficients without artificial averaging and smoothing, the frequency resolution is equal to $\Delta f = \frac{1}{T_d} = \frac{1}{N\Delta t}$. In (1), $\mathbf{Y}_k^{\Re}$ and $\mathbf{Y}_k^{\Im}$ denote the real and imaginary part of $\mathbf{Y}_k$, respectively. The discrete estimator of the PSD matrix of $\mathbf{y}(n)$ is defined as

$$\mathbf{S}_{\mathbf{y}_k} = \mathbf{Y}_k\mathbf{Y}_k^* \tag{2}$$

where $*$ denotes the conjugate transpose of a complex vector. In this work, all 'k' shown in the bracket, in the subscript or in the superscript denote frequency $\omega_k = 2\pi f_k$.A scalar transmissibility function $T_{i,o}^{(k)}$ is defined as the ratio of an arbitrary response $Y_i^{(k)}$ to a reference response $Y_o^{(k)}$, i.e.

$$T_{i,o}^{(k)} = \frac{Y_i^{(k)}}{Y_o^{(k)}} \tag{3}$$

For the special case with a single force $F_j(\omega_k)$ applied at the $j-th$ dof of a dynamic system, scalar transmissibility function reduces to [8,9]

$$T_{i,o}^{(k)} = \frac{Y_i^{(k)}}{Y_o^{(k)}} = \frac{H_{i,j}(\omega_k)F_j(\omega_k)}{H_{o,j}(\omega_k)F_j(\omega_k)} = \frac{H_{i,j}(\omega_k)}{H_{o,j}(\omega_k)} \tag{4}$$