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Feedback linearisation of nonlinear vibration problems: A new formulation by the method of receptances

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ABSTRACT

New output feedback-linearisation theory is presented for the treatment of nonlinear vibration problems by a receptance-based approach. An important aspect is a new formulation for investigating the stability of the zero dynamics. The overall methodology possesses the usual benefits of the receptance method, namely that the system matrices (with associated assumptions and approximations) do not have to be known. In addition, it has the distinction of not requiring the form and parameter values of the nonlinearity when the input and output degrees of freedom are away from the nonlinearity itself. This represents a valuable advance over the conventional time-domain feedback linearisation approach.

1. Introduction

One of the advantages of the receptance method in active vibration control is that it makes use of data readily available from a standard modal test on an engineering structure and does not require knowledge or evaluation of the system matrices $\mathbf{M}, \mathbf{C}, \mathbf{K}$ representing mass, damping and stiffness. It was introduced for single-input systems in 2007 by Ram and Mottershead [1] and extended in 2013 [2] for the multiple-input-multiple-output partial assignment of system eigenvalues. Being a frequency domain approach, the method has been quite widely applied to linear time-invariant (LTI) systems. Ghandchi Tehrani et al. [3] applied the receptance-based method to demonstrate successfully considerable modification of the dynamic behaviour of a heavy modular test structure using electromagnetic actuators and accelerometers. The same authors developed a receptance method for robust eigenvalue assignment [4]. Singh and his colleagues [5–7] applied it to problems of flutter control in aeroelastic systems and Ouyang et al. [8] developed a receptance method with convex constraints, thereby ensuring the existence of a unique solution with a fast-converging algorithm. Bai et al. [9] introduced a robust partial pole placement method, including time delay, based on a combination of receptances and system matrices.

Application of the receptance method to nonlinear vibration control seems to be restricted to the work of Ghandchi Tehrani et al. [10] who made use of the describing function approximation [11] whereby the functional nonlinearity is replaced by amplitude dependence. They considered the single degree of freedom Duffing oscillator and assigned the peak resonance to a prescribed value. The nonlinear controls literature [12–14] is concentrated mainly upon time-domain methods, including sliding mode control, backstepping and feedback linearisation. The output-linearisation problem is that of the under-actuated system with equal numbers, m, of actuators and sensors less than the number, n, of degrees of freedom

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of the system i.e. $m < n$. There is no requirement for actuators and sensors to be placed at the same locations. Coordinate transformation is carried in order to achieve two systems of nonlinear equations that can be treated separately. The first system is controllable and the control input is designed to cancel the nonlinearity, resulting in a linear system with $2m$ eigenvalues which may be assigned to desired locations on the s-plane. The second system of $(n-m)$ equations, which are generally nonlinear, are designed to be uncontrollable. Assignment of the $2m$ eigenvalues is necessary but not sufficient for stability of the complete system. In order to achieve the required full-system stability the zero-dynamics - obtained by setting the output coordinates to zero in the uncontrollable $(n-m)\times n$ system - must also be stable. This is the equivalent of the minimum phase controller in LTI systems. There are numerous papers describing the application of conventional time-domain feedback linearisation, including Fossen and Paulsen [15] on ship steering, Ko et al. [16] on aeroelastic wind-tunnel tests, Poursamand [17] on the control of antilock brakes, Bechlioulis and Rovithakis [18] on multiple-input multiple-output (MIMO) tracking controllers, Shojaei et al. [19] on tracking control for wheeled robots, and Tuan et al. [20] on the nonlinear dynamics of overhead cranes. The present authors' research includes the development of feedback linearisation methods for the treatment of non-smooth nonlinearity (bilinear stiffness and freeplay) [21] and wind-tunnel aerofoil tests with structural nonlinearity [22]. Preliminary experimental research on non-smooth nonlinearity using a laboratory 3-degree of freedom mass-spring system [23] demonstrated the practical feasibility of the feedback linearisation approach and is presently being further developed, including the practical application of the method described (in theory) in the present article.

In Section 2 we present new theory for investigating the zero dynamics of LTI systems using receptances, whereas in Section 3 the conventional matrix-based procedure is described. Then, in Section 4, the new theory is validated by showing that the two approaches produce identical results. In the analysis, we represent the nonlinearity using describing functions, so that for sinusoidal displacement amplitudes maintained constant, the system is truly LTI. This is equivalent to a slowly progressing amplitude-controlled swept sine test. The usual requirement for stability of the zero dynamics is then a straightforward eigenvalue problem. The use of describing functions, though suitable for many structural dynamics applications, is limited to the analysis of weak nonlinearities so that the stability obtained is only locally asymptotically convergent to a stable-zero attractor. In Section 5 new theory is presented for output feedback linearisation using receptances. The methodology described has the same advantage as the receptance method in LTI systems, namely that the system matrices, which generally involve approximation and inaccuracy, are not required, and instead are replaced by measured receptance data. Also, when the input and output degrees of freedom are away from the nonlinearity, it is unnecessary to know the form and parameters of the nonlinearity – the only exception being its location, which must be known. This is a considerable advantage over the conventional time-domain feedback linearisation approach where the form of the nonlinearity must be known and any error in its parameterisation must be corrected adaptively. The theory is supported by a series of numerical examples.

2. Zero dynamics for the LTI system using receptances

Consider the linear $n\times n$ system with m inputs and m outputs $(m < n)$,

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{B}\mathbf{u} \\ &\mathbf{y} = \mathbf{x}_{1:m} = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{1}$$

where $\mathbf{M},\mathbf{C},\mathbf{K}\in\mathbb{R}^{n\times n}$ are the mass, damping, and stiffness matrices, respectively; $\mathbf{B}\in\mathbb{R}^{n\times m}$ is the force distribution matrix. The vector of input forces is defined by $\mathbf{u}\in\mathbb{R}^{m\times 1}$; the system displacements are given by $\mathbf{x}\in\mathbb{R}^{n\times 1}$ and the available outputs are $\mathbf{y}\in\mathbb{R}^{m\times 1}$. Expressing Eq. (1) in frequency domain yields,

$$\begin{aligned} &\mathbf{x}(s) = \mathbf{H}(s)(\mathbf{B}\mathbf{u}(s)) \\ &\mathbf{y}(s) = \mathbf{x}_{1:m}(s) \end{aligned} \tag{2}$$

where $\mathbf{H}(s) = (\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K})^{-1}\in\mathbb{C}^{n\times n}$ is the receptance matrix.

A transformation matrix may be defined as,

$$\begin{aligned} \mathbf{z} &= \mathbf{T}\mathbf{x} \\ \mathbf{T} &= \begin{bmatrix} \mathbf{I}_{m\times m} & \mathbf{0}_{m\times(n-m)} \\ \multicolumn{2}{c}{\mathbf{V}_{(n-m)\times n}} \end{bmatrix} \end{aligned} \tag{3}$$

where

$$\begin{aligned} \mathbf{V}\mathbf{B} &= \mathbf{0} \\ \mathbf{V}\mathbf{V}^T &= \mathbf{I}_{(n-m)\times(n-m)} \end{aligned} \tag{4}$$