Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 292 (2015) 138–170
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

# Partitioning strategies for the interaction of a fluid with a poroelastic material based on a Nitsche's coupling approach

M. Bukač[a], I. Yotov[b], R. Zakerzadeh[c], P. Zunino[c,*]

[a] Department of Applied and Computational Mathematics and Statistics, University of Notre Dame, IN 46556, USA
[b] Department of Mathematics, University of Pittsburgh, PA 15260, USA
[c] Department of Mechanical Engineering & Materials Science, University of Pittsburgh, PA 15261, USA

Available online 25 November 2014

**Abstract**

We develop a computational model to study the interaction of a fluid with a poroelastic material. The coupling of Stokes and Biot equations represents a prototype problem for these phenomena, which feature multiple facets. On one hand, it shares common traits with fluid–structure interaction. On the other hand it resembles the Stokes–Darcy coupling. For these reasons, the numerical simulation of the Stokes–Biot coupled system is a challenging task. The need of large memory storage and the difficulty to characterize appropriate solvers and related preconditioners for the equations at hand are typical shortcomings of classical discretization methods applied to this problem, such as the finite element method for spatial discretization and finite differences for time stepping. The application of loosely coupled time advancing schemes mitigates these issues, because it allows to solve each equation of the system independently with respect to the others, at each time step. In this work, we develop and thoroughly analyze a loosely coupled scheme for Stokes–Biot equations. The scheme is based on Nitsche's method for enforcing interface conditions. Once the interface operators corresponding to the interface conditions have been defined, time lagging allows us to build up a loosely coupled scheme with good stability properties. The stability of the scheme is guaranteed provided that appropriate stabilization operators are introduced into the variational formulation of each subproblem. The error of the resulting method is also analyzed, showing that splitting the equations pollutes the optimal approximation properties of the underlying discretization schemes. In order to restore good approximation properties, while maintaining the computational efficiency of the loosely coupled approach, we consider the application of the loosely coupled scheme as a preconditioner for the monolithic approach. Both theoretical insight and numerical results confirm that this is a promising way to develop efficient solvers for the problem at hand.


*Keywords:* Fluid–structure interaction; Poroelasticity; Operator-splitting scheme; Nitsche's method; Preconditioning

## 1. Introduction

Several phenomena in nature involve the interaction of a free fluid with a porous and deformable material. In geomechanics, the behavior of aquifers and groundwater flow is significantly influenced by the interaction of

* Authors are listed in alphabetical order. Corresponding author. Tel.: +1 412 624 9774.
*E-mail addresses:* mbukac@nd.edu (M. Bukač), yotov@math.pitt.edu (I. Yotov), raz25@pitt.edu (R. Zakerzadeh), paz13@pitt.edu (P. Zunino).

http://dx.doi.org/10.1016/j.cma.2014.10.047
0045-7825/

deformation and flow at the pore scale. In the same context, the efficient extraction of oil and gas from the subsurface relies on the ability to predict and control these phenomena. Another example from biology is the perfusion of living tissues, where the fluid carried by the main vessels is distributed by filtration to the surrounding material. All these phenomena are characterized by some common traits. In particular, they share a multiphysics nature, because they are governed by the interaction of fluid and solid mechanics.

In the general framework of continuum mechanics, several models are available to describe these phenomena. To model the free fluid, we consider for simplicity the Stokes equations, under the assumptions of incompressible and Newtonian rheology. A well accepted model for characterizing the behavior of a poroelastic material is provided by the Biot equations. The Biot system consists of the governing equations for the deformation of an elastic skeleton, which is completely saturated with fluid. The average velocity of the fluid in the pores is modeled using the Darcy equation, complemented with an additional term that depends on the volumetric deformation of the porous matrix. Indeed, this term accounts for the poroelastic coupling. In this work we focus on the coupling of the Stokes and Biot models, for phenomena where time and space dependence of the unknowns play a significant role.

The numerical discretization of the problem at hand features several difficulties. Loosely coupled schemes for fluid–structure interaction (FSI) may turn out to be unconditionally unstable, under a particular range of the physical parameters of the model [1,2]. This is the so called *added-mass effect*. Furthermore, accounting for a poroelastic material model requires addressing issues typical of partitioned methods for flows [3–5] arising in particular for the coupling of Stokes–Darcy models, which represents the paradigm for studying the interaction of free flows with subsurface filtration [6–13]. An additional difficulty is combining the Eulerian description of the moving fluid domain with the typical Lagrangian parametrization of the structure [14,15]. In this work we consider only fixed domains $\Omega_f$ and $\Omega_p$ representing the reference (Lagrangian) configuration of the fluid and solid domains, respectively. This is a common approach for fluid–structure interaction problems, see for example [16], that is adopted here to simplify the complexity of the *fluid–structure–porous* interaction. Although simplified, this problem still retains the main difficulties associated with the added-mass effect and with the fluid–porous media coupling. As a consequence of the fixed domain assumption, the computational model that we propose is suitable in the range of small displacements. We note that in recent work [17] we have studied the differences between the fixed and the moving domain approaches. The results, obtained for a slightly different test problem than the one considered in Section 5, including the effect of an elastic membrane at the interface of the fluid with the thick structure, indicate that in this particular case the effect of *geometric* and *convective* nonlinearities is negligible.

The above observations suggest that the solution of the time dependent Biot–Stokes coupled equations is challenging from both the theoretical and computational standpoints. Concerning the analysis, the coupled problem and in particular the formulation of appropriate interface conditions has been studied in [18–20]. From the computational viewpoint, the multiple facets of the problem have been addressed in several studies. For example, the coupling of subsurface flow and geomechanics have been recently addressed in [21–23]. Depending on the field of application, different formulations are available to couple a free flow with a saturated poroelastic material. In the context of geosciences, this coupled problem is used to model the interaction of the material with fractures, as in [24–28,19]. In the context of biomedical applications, FSI studies involving poroelastic materials are scanty. Among the available contributions, we mention [29,30], as well as our recent work [31].

The objective of this work is to develop and analyze a loosely coupled numerical solver for the coupled Biot–Stokes system. More precisely, we design a time advancing scheme which allows us to independently solve the governing equations of the system at each time step. Resorting to time splitting approaches mitigates the difficulty to identify appropriate solvers for the coupled system and reduces the need of large memory storage. The main drawback of loosely coupled splitting schemes is possible lack of stability and accuracy. To overcome these natural limitations, we adopt a non-standard approach for the approximation of the coupling conditions, which is inspired by Nitsche's method for the enforcement of boundary conditions, and it consists of adding appropriate interface operators to the variational formulation of the problem. Using time-lagging, the variational coupled problem can be split into three independent subproblems involving the elasticity equation, Darcy equation for flow in porous media and the Stokes problem, respectively. The stability analysis of the resulting scheme shows how to design appropriate stabilization terms that guarantee the stability of the time advancing algorithm. The Nitsche's coupling approach allows for treating the mixed form of Darcy flow and thus provides accurate approximation to the filtration velocity. This is an alternative to the Lie-splitting scheme developed in [31], which is suitable for the pressure formulation of Darcy flow.