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Degradation data analysis based on a generalized Wiener process subject to measurement error

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A B S T R A C T

Wiener processes have received considerable attention in degradation modeling over the last two decades. In this paper, we propose a generalized Wiener process degradation model that takes unit-to-unit variation, time-correlated structure and measurement error into considerations simultaneously. The constructed methodology subsumes a series of models studied in the literature as limiting cases. A simple method is given to determine the transformed time scale forms of the Wiener process degradation model. Then model parameters can be estimated based on a maximum likelihood estimation (MLE) method. The cumulative distribution function (CDF) and the probability distribution function (PDF) of the Wiener process with measurement errors are given based on the concept of the first hitting time (FHT). The percentiles of performance degradation (PD) and failure time distribution (FTD) are also obtained. Finally, a comprehensive simulation study is accomplished to demonstrate the necessity of incorporating measurement errors in the degradation model and the efficiency of the proposed model. Two illustrative real applications involving the degradation of carbon-film resistors and the wear of sliding metal are given. The comparative results show that the constructed approach can derive a reasonable result and an enhanced inference precision.

1. Introduction

Assessing life information is essential to improve a product's quality and reliability. In the traditional reliability research, life tests that record only time-to-failure data are typically employed to evaluate the population reliability information for identical products. For highly reliable and long life products, however, it is difficult to estimate the reliability via a life testing method because very few or even no failure will probably occur within a reasonable time period. Although accelerated life tests can be considered in this situation, the acceleration mechanism may not properly repeat the actual failure process [1]. Thereafter, an alternative approach is to observe the product's degradation. In many practical situations, there exist performance characteristics that degrade over time such that a failure will occur when the amount of degradation for a unit reaches a certain critical level. Then the degradation modeling can offer an approach to trace the underlying deterioration

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process and assess the reliability. Therefore, the degradation analysis has received widespread attention from researchers and engineers, and has become a popular technique in statistics and reliability [2,3].

It is well recognized that degradation processes often illustrate a complex mechanism and uncertainty over time. Thus, stochastic process formulations are frequently adopted to characterize the evolution of degradation processes [4]. Singpurwalla [5] provided an overview on these stochastic processes for describing failure mechanisms, such as Markov chain, Wiener process and Gamma process. Among them, Wiener process is one of the most prominent degradation models and has been studied rather extensively due to its mathematical properties and physical interpretations. By assuming the mean degradation path is linear or can be linearized, the Wiener degradation process with a linear drift has been studied by Ye et al. [6], Peng et al. [7], Tsai et al. [8], Guo et al. [9], and Park et al. [10]. However, not all the nonlinear degradation processes can be properly linearized, then the dynamics of the degradation processes cannot be accurately characterized by linear degradation models in such a situation [11,12]. Inspired by this problem, a nonlinear Wiener process has been investigated by Wang et al. [13] and Si et al. [14], but the variance of the nonlinear Wiener process is still linear over time. To remedy this deficiency, a well-adopted form for the regular Wiener process was presented as [15]

$$X(t) = \beta\Lambda(t) + \sigma B(\Lambda(t)) \tag{1}$$

where $B(\cdot)$ denotes a standard Wiener process, and $\Lambda(t)$ is defined as the transformed time scale. The heterogeneous Wiener process $X(t)$ has s-independent and normally distributed increments; viz. $\Delta X(t) = X(t+\Delta t) - X(t)$ is s-independent of $X(t)$, $X(t) \sim N(\beta\Lambda(t), \sigma^2\Lambda(t))$ and $\Delta X(t) \sim N(\beta\Lambda(t+\Delta t) - \beta\Lambda(t),\ \sigma^2\Lambda(t+\Delta t) - \sigma^2\Lambda(t))$. The Wiener process expressed by Eq. (1) has been widely applied in degradation data analysis. Whitmore [15] used a time-transformed Wiener process to model the performance deterioration of the transistor. Whitmore and Schenkelberg [16] presented a power law time-transformed Wiener process to capture the degradation of self-regulating heating cables. Tseng and Peng [17] adopted an exponential time-transformed Wiener process to describe the deterioration path of a specific light emitting diode (LED) product. Motivated by the wear of magnetic heads of hard disk drives, Ye et al. [18] investigated a mixed effect Wiener process with a power law time-transformed scale to capture the heterogeneity within the population.

From these time-transformed Wiener process models, it should be noted that certain relationship, also can be called limitation, exists between mean and variance because of the one time-transformed $\Lambda(t)$. The mean of the degradation process is constantly proportional to its variance over time. In many practical engineering, however, this relationship may not be realistic. Taking the linear transformed time scale situation (i.e., $\Lambda(t)$ is a linear function of test time) as an example, we should note that the time-transformed Wiener process will illustrate both linear trend assuming $\beta\Lambda(t)$ and variance $\sigma^2\Lambda(t)$. Although for the commonly used mixed effects model situation $\beta \sim N(\mu, \kappa^2)$, the Wiener process will exhibit a linear mean $\mu\Lambda(t)$ and a quadratic variance $\kappa^2\Lambda^2(t) + \sigma^2\Lambda(t)$, this quadratic variance is not a generalized one because of the only one transformed time scale $\Lambda(t)$.

To settle this problem, Tseng et al. [19] proposed a generalized Wiener process model for degradation analysis with two transformed time scales $\Lambda(t)$ and $\tau(t)$

$$X(t) = \beta\Lambda(t) + \sigma B(\tau(t)) \tag{2}$$

It can be concluded that Eq. (2) is an improved degradation model comparing with Eq. (1), because the previously mentioned correlation limitation between mean and variance of the time-transformed Wiener process can be broken through. However, Tseng et al. [19] restricted their attention to the case $\Lambda(t) = \tau(t)$ in their study, because they concluded that it was rather difficult to obtain an explicit form of the lifetime distribution when $\Lambda(t) \neq \tau(t)$. Therefore, they only proposed a general Wiener process model form with two transformed time scales without any essential studies or applications. Subsequently, Wang et al. [12,20] investigated the issue of real-time reliability evaluation based on the generalized Wiener process model and derived an approximate form of the PDF of the failure time under a mild assumption. In our previous work [21], we constructed a Wiener process model which can be considered as a special case of the generalized two-transformed time scale Wiener model expressed by Eq. (2), where drift β and diffusion σ are both constants, $\Lambda(t) = a + bt$ and $\tau(t) = dt^2$.

In real applications, however, it is inevitable that measurement errors are introduced during the observation process because of the imperfect instruments, procedures and random environments in practical engineering. Linear Wiener processes with measurement errors have been widely studied in the literature. Motivated by a laser deterioration problem, Peng et al. [7] proposed a linear degradation model which allows us to take "unit-to unit variation", "time-correlated structure", and "measurement error" into considerations simultaneously. Pan et al. [22] utilized a linear Wiener model to capture the degradation of photovoltaic products and discussed the effects of measurement error on model parameter estimation. Tang et al. [23] used the linear model to predict the remaining useful life for lithium-ion batteries. For nonlinear situations, Tang et al. [24] developed a nonlinear Wiener degradation process with measurement errors to implement remaining useful lifetime estimation. In addition, a time-transformed Wiener process with measurement errors has been investigated in Refs. [15,18].

From the above literature, we can observe that the Wiener process has been widely adopted to model the degradation of products. However, a generalized Wiener process with measurement errors, which includes two transformed time scales, has not been thoroughly studied. Moreover, by analyzing the real degradation of carbon-film resistors and a particular metal alloy, we found that the degradation model may be mis-specified without proper consideration of measurement errors. Thus, from the viewpoint of both theory and engineering, the main objective of this paper is to develop a generalized Wiener