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Finite element model updating of concrete structures based on imprecise probability

S. Biswal, A. Ramaswamy *

Civil Engineering Department, Indian Institute of Science, Bangalore 560012, India

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ABSTRACT

Imprecise probability based methods are developed in this study for the parameter estimation, in finite element model updating for concrete structures, when the measurements are imprecisely defined. Bayesian analysis using Metropolis Hastings algorithm for parameter estimation is generalized to incorporate the imprecision present in the prior distribution, in the likelihood function, and in the measured responses. Three different cases are considered (i) imprecision is present in the prior distribution and in the measurements only, (ii) imprecision is present in the parameters of the finite element model and in the measurement only, and (iii) imprecision is present in the prior distribution, in the parameters of the finite element model, and in the measurements. Procedures are also developed for integrating the imprecision in the parameters of the finite element model, in the finite element software Abaqus. The proposed methods are then verified against reinforced concrete beams and prestressed concrete beams tested in our laboratory as part of this study.

1. Introduction

Condition assessment of structures comprises of both experimental and numerical methods. In experiments, structural responses are measured for known loads, geometric properties and boundary conditions. In numerical methods, finite element (FE) models are constructed from available drawings or from field measurements on the prototype and the measured structural responses from the experiments for known loads are used to update the physical parameters of the FE model until a good agreement is achieved between the responses estimated from the numerical model and the actual measurements. A detailed discussion on FE model updating techniques is given in [1–3]. However in reality uncertainties are present in the form of modeling uncertainties and the measurement uncertainties. Modeling uncertainties occur due to constructing a representative model of the real structure through finite element modeling, and representing the current state of the structure through changes in material parameters of the finite element model. Measurement uncertainties are always present in the measurements despite the accuracy with which these are measured or the precision of the instruments used for the measurement [4]. The sources of error in the measurements on concrete structures coming from displacement, strain and acceleration measurements using displacement gauges, strain gauges and accelerometers can be divided into systematic errors

* Corresponding author.
E-mail addresses: skbiswal@cistup.iisc.ernet.in (S. Biswal), ananth@civil.iisc.ernet.in (A. Ramaswamy).

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and random errors. Systematic errors include gauge sensitivities, calibration accuracies, amplitude linearities and temperature corrections to the gauge sensitivities, which are given in terms of plus-minus ranges, and the round off errors in the measured responses, which are better represented by interval bounds. All the errors apart from the systematic errors are called random errors. Some of the sources of random errors are known (error due to digitization from analog to digital and vice versa, error in transmission, aliasing, leakage, windowing and signal processing, and errors inherent in measuring instruments), but some are unknown, and these kind of errors cannot be eliminated completely.

Traditionally Bayesian methods are used to quantify the uncertainties in the FE model parameters through estimated posterior distribution functions, when uncertainties in the measurements are given through random distributions. Updating models and the associated uncertain parameters in a Bayesian statistical framework is given in [5–7]. However as mentioned in [8] probabilistic arithmetic has limitations with handling the representation error, the dependency error, and the implementation error. Also *Bayes rule doesn't tell you anything you didn't already believe* [9] i.e. setting the prior distribution determines the posterior distribution. For parameter estimation problems the probabilistic approaches need such a large amount of data for modeling uncertainty present in the damage parameters that in many cases it is difficult to choose the prior distribution.

Intervals can be used to model uncertainty of variables when the evidence about their probability distributions are not known. Set inversion via interval analysis is used in [10] for the guaranteed bounded estimation of uncertain parameters. However the main problem in their method [10], is that the computational time increases exponentially with the number of parameters to be estimated. Sensitivity based FE model updating procedure with interval valued damage parameters is proposed in [11] for damage identification of structures. The difficulties in the study presented in [11] are the first order approximation in the generation of sensitivity matrix, and the inability of interval analysis in carrying out iterative solution methods to reduce the error caused by the first order approximation.

When the measured signal contains both interval uncertainties (e.g. plus minus type of error) and statistical randomness (e.g. electronic noise), the total uncertainties can be represented through imprecise probabilities, where bounds are provided to the parameters of the distributions assigned to represent the statistical randomness. Imprecise probabilities can be modeled through probability boxes (p-boxes) [12–14] specified by a pair of non-decreasing cumulative distribution functions (CDF's). If $Z = (Z_1, Z_2, \dots Z_m)$ is the measurement vector with probability distribution F_Z, and ξ_Z represents the vector of parameters of the distribution F_Z (such as mean and standard deviation) the interval ranges of ξ_Z are given as $[\underline{\xi}_Z, \overline{\xi}_Z]$, the p-box $[\underline{F}_Z, \overline{F}_Z]$ as given in [15] can be stated as

$$\underline{F}_Z = \min\{F(z, \xi_Z) : \underline{\xi}_Z \leqslant \xi_Z \leqslant \overline{\xi}_Z\} \tag{1}$$

$$\overline{F}_Z = \max\{F(z, \xi_Z) : \underline{\xi}_Z \leqslant \xi_Z \leqslant \overline{\xi}_Z\} \tag{2}$$

The objective of the present study is to estimate the vector of uncertain parameters θ, of the FE model of concrete structures, when the uncertainties in the measurements Z are given through the p-boxes $[\underline{F}_Z, \overline{F}_Z]$. This involves the generation of samples of θ from the posterior $p(\theta|Z)$). Although imprecise probability has been studied in the existing literature [12,14,16–20], its application in the field of uncertainty modeling in civil engineering structures is limited mostly to structural reliability analysis, or in the forward problem of quantifying the uncertainties in the output function when the uncertainties in the input parameters are expressed through imprecise probability.

Imprecise probability based methods are developed in the present study for the parameter estimation in the FE model updating for concrete structures when the measurements are imprecisely defined (through the p-boxes $[\underline{F}_Z, \overline{F}_Z]$). Three different cases are considered (i) imprecision is present in the prior distribution and in the measurements only, (ii) imprecision is present in the parameters of the FE model and in the measurement only, and (iii) imprecision is present in the prior distribution, in the parameters of the FE model, and in the measurements. Procedures are also developed for integrating the imprecision in the parameters of the FE model, into the FE software Abaqus [21]. The proposed methods are then verified using results from reinforced concrete and prestressed concrete beams tested in the laboratory for this study.

2. Parameter estimation with imprecise probability

Imprecise probability methods are based on the belief that it is difficult to specify the prior distribution and the likelihood function, and they can only be specified by a finite set of such functions. Such a set of prior models is selected such that the calculation of range of posterior distribution should be as easy as possible, and that the set should not contain unreasonable priors and should consider as many reasonable priors as possible. A set of distribution functions can be fitted against an interval data set assuming the shape of distribution functions [12]. In these methods all possible combinations of the prior distributions and the likelihood functions result in a set of posterior distributions. The set of such distribution functions can be reduced to probability boxes where they are bounded by the lower and the upper probability distribution in the set. Such representation is much simpler to use than a huge collection of distributions encompassing all possible combinations of scalars within these intervals.