Mechanical Systems and Signal Processing 94 (2017) 279–296

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

# Incomplete data based parameter identification of nonlinear and time-variant oscillators with fractional derivative elements

Ioannis A. Kougioumtzoglou [a,*], Ketson R.M. dos Santos [a], Liam Comerford [b]

[a] Department of Civil Engineering and Engineering Mechanics, Columbia University, 500 W. 120th St., 610 Mudd, New York, NY 10027, USA
[b] Institute for Risk and Reliability, Leibniz University Hannover, Hannover, Germany

ARTICLE INFO

*Article history:*
Received 14 December 2016
Received in revised form 23 February 2017
Accepted 4 March 2017

*Keywords:*
System identification
Nonlinear system
Fractional derivative
Harmonic wavelet
Incomplete data

ABSTRACT

Various system identification techniques exist in the literature that can handle non-stationary measured time-histories, or cases of incomplete data, or address systems following a fractional calculus modeling. However, there are not many (if any) techniques that can address all three aforementioned challenges simultaneously in a consistent manner. In this paper, a novel multiple-input/single-output (MISO) system identification technique is developed for parameter identification of nonlinear and time-variant oscillators with fractional derivative terms subject to incomplete non-stationary data. The technique utilizes a representation of the nonlinear restoring forces as a set of parallel linear sub-systems. In this regard, the oscillator is transformed into an equivalent MISO system in the wavelet domain. Next, a recently developed $L_1$-norm minimization procedure based on compressive sensing theory is applied for determining the wavelet coefficients of the available incomplete non-stationary input-output (excitation-response) data. Finally, these wavelet coefficients are utilized to determine appropriately defined time- and frequency-dependent wavelet based frequency response functions and related oscillator parameters. Several linear and nonlinear time-variant systems with fractional derivative elements are used as numerical examples to demonstrate the reliability of the technique even in cases of noise corrupted and incomplete data.



## 1. Introduction

Research efforts in the field of system identification over the past few decades have focused not only on the development of novel numerical techniques [1–3], but also on deriving conditions and assessing the feasibility of such techniques for effective parameter identification when faced with limited data [4–6]. In general, several major challenges need to be addressed for efficacious system parameter identification.

First, in real-life situations, the statistics of measured/available data most often exhibit a time/space-varying behavior. For instance, most environmental processes/excitations (and subsequently the system responses) can be realistically described as non-stationary stochastic processes, i.e. their statistics (as well as their frequency content potentially) vary with time. Thus, traditional/standard signal processing tools, such as Fourier analysis, are not adequately equipped for analyzing the above signals. In this regard, the short-time Fourier Transform, the Gabor transform, wavelets, chirplets, and the

* Corresponding author.
*E-mail addresses:* ikougioum@columbia.edu (I.A. Kougioumtzoglou), kmd2191@columbia.edu (K.R.M. dos Santos), comerford@bauinf.uni-hannover.de (L. Comerford).

http://dx.doi.org/10.1016/j.ymssp.2017.03.004
0888-3270/

Wigner-Ville distribution [7–9] constitute some of the most popular tools for analyzing the non-stationary spectral content of a signal.

Second, most often there are limited, incomplete and/or missing data due to several reasons, such as cost (e.g. expensive sensor maintenance in harsh conditions/remote areas), frequency and unpredictability of the effect, data loss or corruption (e.g. sensor failures, power outages, etc.), as well as limited bandwidth/storage capacity. In this regard, various existing signal reconstruction techniques (e.g. Lomb-Scargle periodogram, iterative deconvolution method CLEAN, Auto-Regressive-Moving-Average model based techniques, etc.) can be applied that can handle missing data; see [10] for a review. Note, however, that most of the above techniques for signal reconstruction under missing data lack versatility and exhibit certain limitations. Indicatively, relatively strong a priori assumptions about the signal may be required [11]. Also, the techniques can be computationally demanding [12], perform well only in cases of few missing data [13], and may not be applicable for non-stationary signals, at least in a straightforward manner [14].

Third, the need for more accurate materials/media behavior modeling has led recently to utilizing advanced mathematical concepts/tools such as fractional calculus [15,16]. Besides the fact that fractional calculus can be construed as a generalization of classical calculus (and as such provides with enhanced modeling flexibility), it has been successfully employed in engineering mechanics for developing "non-local" continuum mechanics models/theories [17,18], as well as for viscoelastic material modeling [19]. In this regard, note that most system identification techniques have been developed and tailored for treating conventional governing dynamics equations based on traditional/classical continuum (or discrete) mechanics theories. Their generalization for treating systems following a fractional calculus modeling is not at all straightforward.

Overall, various techniques have been developed in the literature that are capable of handling either non-stationary data [20,21], or missing data cases [10], or systems with fractional derivative terms [22]. However, to the best of the authors' knowledge, there are not many (if any) system identification techniques that can address all three aforementioned challenges simultaneously in a consistent manner. In this regard, a novel multiple-input/single-output (MISO) system identification technique is developed herein that utilizes a representation of the nonlinear restoring forces as a set of parallel linear sub-systems, and relies on knowledge of measured excitation-response data. The original technique was developed by Bendat and co-workers [23–25] and has found applications in various diverse fields [26–29]. It was generalized recently by Kougioumtzoglou and Spanos [30] based on harmonic wavelets to account for non-stationary inputs and time-varying system parameters. In this paper, the technique is further extended to account for incomplete/missing data as well as for systems with fractional derivative elements. Several linear and nonlinear time-variant systems with fractional derivative elements are used as numerical examples to demonstrate the reliability of the technique even in cases of noise corrupted and incomplete data.

## 2. Challenges: non-stationary and incomplete data

### 2.1. Non-stationary data: A harmonic wavelet treatment

The family of generalized harmonic wavelets (GHW), proposed by Newland [31], utilizes two parameters $(m,n)$ for the definition of the bandwidth at each scale. One of its main advantages relates to the fact that these two parameters, in essence, decouple the time–frequency resolution achieved at each scale from the value of the central frequency. Further, harmonic wavelets have proven to be particularly useful for structural dynamics related applications [20] due to their non-overlapping, box-shaped frequency spectrum, their orthogonality properties, and the convenience of combining harmonic balance with statistical linearization techniques [30,32,33]. Indicative recent system identification techniques and applications can be found in [34–36]. The GHW has a band-limited, box-shaped frequency spectrum in the frequency domain. A wavelet of $(m,n)$ scale and $(k)$ position in time attains a representation in the frequency domain of the form

$$\Psi^{G}_{(m,n),k}(\omega) = \begin{cases} \frac{1}{(m-n)\Delta\omega} \exp\left(-i\omega \frac{kT_0}{m-n}\right), & m\Delta\omega \leqslant \omega \leqslant n\Delta\omega m, \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where $T_0$ is the duration of the signal, $\omega$ is the frequency in *rad/s*, $i = \sqrt{-1}$ is the imaginary unit, $\Delta\omega = \frac{2\pi}{T_0}$, and $m\Delta\omega \leqslant \omega \leqslant n\Delta\omega$ is the bandwidth of the box-shaped spectrum. The time domain representation of the GHW is obtained by taking the Fourier transform of Eq. (1), i.e.,

$$\psi^{G}_{(m,n),k}(t) = \frac{\exp\left(in\Delta\omega\left(t - \frac{kT_0}{n-m}\right)\right) - \exp\left(im\Delta\omega\left(t - \frac{kT_0}{n-m}\right)\right)}{i(n-m)\Delta\omega\left(t - \frac{kT_0}{n-m}\right)}. \tag{2}$$

Next, the continuous generalized harmonic wavelet transform (GHWT) for a signal $f(t)$ is given by [31]

$$W^{G}_{(m,n),k}[f] = \frac{n-m}{T_0} \int_{-\infty}^{\infty} f(t)\bar{\psi}^{G}_{(m,n),k}(t)dt, \tag{3}$$

whereas the inverse transform allows for the exact reconstruction of the target signal in the form