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A multi-step method for partial eigenvalue assignment problem of high order control systems ☆

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ABSTRACT

In this paper, we consider the partial eigenvalue assignment problem of high order control systems. Based on the orthogonality relations, we propose a new method for solving this problem by which the undesired eigenvalues are moved to desired values and keep the remaining eigenvalues unchanged. Using the inverse of Cauchy matrix, we give the solvable condition and the explicit solutions to this problem. Numerical examples show that our method is effective.

1. Introduction

Consider the high order control system

$$M_k z^{(k)} + M_{k-1} z^{(k-1)} + \cdots + M_1 \dot{z} + M_0 z = Bu, \tag{1}$$

where $M_j \in R^{n\times n}$ $(j = 0, 1, \ldots, k)$ are system coefficient matrices and M_k is nonsingular, $B \in \mathbf{R}^{n\times m}$ is the full rank control matrix and $u \in \mathbf{R}^m$ is the control vector. This kind of control systems arises mainly in vibration control[1]. If $m = 1$, the system (1) reduces the single-input control system, and if $m > 1$, it is the multi-input control system.

To combat undesirable effects of vibrations, such as resonance, caused by a few undesired eigenvalues of the systems, one needs to reassign those few eigenvalues, leaving the rest unchanged. One of the vibration control methods is state feedback control, namely, the control vector u is chosen as

$$u = F_{k-1}^\top z^{(k-1)} + F_{k-2}^\top z^{(k-2)} + \cdots + F_0^\top z, \tag{2}$$

where $F_{k-j} \in R^{n\times m}$ $(j = 1, \ldots, k)$ are state feedback matrices, such that the closed-loop system

$$M_k z^{(k)} + (M_{k-1} - BF_{k-1}^\top) z^{(k-1)} + \cdots + (M_1 - BF_1^\top)\dot{z} + (M_0 - BF_0^\top) z = 0, \tag{3}$$

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has desired properties. Mathematically, the partial eigenvalue assignment problem for the high order control systems is that of finding the matrices M_j $(j = 0, 1, \ldots, k)$ such that a few eigenvalues of the closed-loop pencil

$$H_c(\lambda) = \lambda^k M_k + \lambda^{k-1}(M_{k-1} - BF_{k-1}^\top) + \cdots + \lambda(M_1 - BF_1^\top) + (M_0 - BF_0^\top) \tag{4}$$

are replaced by suitably chosen ones, keeping the remaining eigenvalues unchanged, i.e. the no spill-over property is preserved. See [2–5]. This leads to the partial eigenvalue assignment problem of high order systems (PEAP-HOS).

Problem PEAP-HOS. Given the $n \times n$ real matrices $M_j \in R^{n\times n}$ $(j = 0, 1, \ldots, k)$ with M_k nonsingular, the full rank control matrix $B \in \mathbf{R}^{n\times m}$, and give the self-conjugate subset $\{\lambda_i\}_{i=1}^p (p < kn)$ of the open-loop spectrum $\{\lambda_i\}_{i=1}^{kn}$ and the corresponding left eigenvector set $\{y_i\}_{i=1}^p$ and a self-conjugate set $\{\mu_i\}_{i=1}^p$, find the state feedback matrices $F_{k-j} \in R^{n\times m}$ $(j = 1, 2, \ldots, k)$ such that the closed-loop pencil

$$H_c(\lambda) = \lambda^k M_k + \lambda^{k-1}(M_{k-1} - BF_{k-1}^\top) + \cdots + \lambda(M_1 - BF_1^\top) + (M_0 - BF_0^\top), \tag{5}$$

has the desired eigenvalues $\{\mu_i\}_{i=1}^p$, and the eigenpairs $\{\lambda_i, x_i\}_{i=p+1}^{kn}$.

In 2007, Ram and Mottershead [6] first proposed the receptance method in active vibration control. This method can assign the expected eigenvalues and keep the no spillover property. Receptances are readily available from model tests whereas the system matrices are often unknown, can be approximated for example by using finite elements, but there are always modelling assumptions, and validated models are expensive to achieve. Applying the receptance method to high order systems, Zhang [7] developed this method for solving Problem PEAP-HOS with time delay. For developments of the receptance method, see [8–12].

In [13], Mao proposed a method for solving the minimum norm partial eigenvalue assignment problem of high order systems. Cai et al. [14] considered the robust eigenvalue assignment problem of high order systems. For second order systems, Liu and Yuan [15] proposed a multi-step method for solving the partial eigenvalue problem with time delay. In this paper, we extend the second order control systems to the high order control systems, and propose a multi-step method for solving Problem PEAP-HOS. Based on the orthogonality relations and the inverse of Cauchy matrix, we give the solvable condition and the explicit solutions to this problem.

Throughout this paper, we use the following assumptions.

(1) the system $(\{M_j\}_{j=0}^k, B)$ is partially controllable with respect to $\{\lambda_i\}_{i=1}^p$;
(2) $\{\lambda_i\}_{i=1}^p \neq \emptyset, \{\mu_i\}_{i=1}^p \bigcap \{\lambda_i\}_{i=1}^{kn} = \emptyset, \{\lambda_i\}_{i=1}^p \bigcap \{\lambda_i\}_{i=p+1}^{kn} = \emptyset$;
(3) $\lambda_1, \ldots, \lambda_p; \mu_1, \ldots, \mu_p$ are all distinct.

The following notations will be used. The kn eigenvalues of the open-loop system (1) are $\lambda_1, \lambda_2, \ldots, \lambda_{kn}$, and corresponding right eigenvectors are $x_1, x_2, \ldots, x_{kn}$ and left eigenvectors are $y_1, y_2, \ldots, y_{kn}$, and we let

$\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_{kn})$, whose diagonal elements are eigenvalues of the open-loop systems.
$\Lambda_1 = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$, whose diagonal elements are the eigenvalues to be altered.
$\Lambda_2 = diag(\lambda_{p+1}, \lambda_{p+2}, \ldots, \lambda_{kn})$, whose diagonal elements are the eigenvalues kept unchanged.
$\Lambda_{c1} = diag(\mu_1, \mu_2, \ldots, \mu_p)$, whose diagonal elements are the eigenvalues to be assigned.
$Y_1 = [y_1, y_2, \ldots, y_p]$, whose columns are corresponding left eigenvectors of the open-loop systems.
$X = [x_1, x_2, \ldots, x_{kn}]$, whose columns are corresponding right eigenvectors of the open-loop systems.
$X_1 = [x_1, x_2, \ldots, x_p]$.
$X_2 = [x_{p+1}, x_{p+2}, \ldots, x_{kn}]$.

This paper is organized as follows. In Section 2, we give some preliminaries about Problem PEAP-HOS. In Section 3, we give the solution of Problem PEAP-HOS by single-input state feedback control, and the multi-input control is provided in Section 4. The illustrative numerical examples are also reported.

2. Preliminaries

In this section, we state a well-known result on the existence and uniqueness of solution of the partial eigenvalue assignment problem. The notion of controllability is crucial to these results.

Definition 1. [16] The system (1) is controllable, if for any initial state $z(0) = z_0$ and final state z_f, there exists an input $u(t)$ such that the solution t_f satisfies $z(t_f) = z_f$.

Theorem 1. [17] *The system (1) is controllable, then*

$$rank([H(\lambda), B]) = n,$$

for any complex number λ, where $H(\lambda) = \lambda^k M_k + \lambda^{k-1} M_{k-1} + \cdots + \lambda M_1 + M_0$.