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Analytical approach to calculate bending, longitudinal and torsional local stiffness of an asymmetric circumferential crack with contact condition

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ABSTRACT

In this paper, bending, longitudinal and torsional stiffness of an eccentric circumferential crack is investigated with taking into account contact condition on the crack surfaces based on fracture mechanics. Although several researches have analyzed stress intensity factors of symmetric circumferential crack, the stiffness of an asymmetric circumferential crack in different directions (along and perpendicular to eccentricity) regarding contact condition has not been studied by an analytical method until now. In this paper we aim to describe behavior of eccentric circumferential crack under axial loading and establish a relation between axial force and the resulting displacement vector. The twisting angle of asymmetric circumferential crack due to torsional loading is also calculated and compared to twisting angle of a symmetric crack. In order to simulate the local bending stiffness in the contact condition, nonlinear governing equations of bending stiffness associated to cracked beam section is developed by dividing it to strip elements and utilizing stiffness equations related to noncontact condition. It is validated by 3D finite element (FE) nonlinear model. Results show a significant compatibility between presented analytical and 3D FE methods. Moreover results of simulations show that without taking into account contact condition, axial, torsional and bending stiffness of symmetric and asymmetric circumferential crack are equal and radius of un-cracked area is the only influential factor.

1. Introduction

Cracks frequently appear in rotating machinery due to manufacturing flaws or cyclic loading. Hence, researchers have highly focused on this issue to find an efficient approach for observing this problem. Indeed, crack induces local flexibilities, and this significantly alters the vibrational behavior of the rotor. Thus, it is essential to present accurate and reliable model of cracks which could be further used for the crack detection and diagnostic in rotating machinery. The vibration analysis of cracked rotating shafts, including rotating system modeling, crack modeling and processing of vibration signals is one of the most important ways to identify cracks in rotating shafts.

Since 1980s, several researchers have investigated cracks in rotating shaft and published more than thousands of papers in this area [1]. These numerous papers imply that cracks in rotating shafts in terms of crack front shape are divided into several types. Among various types of cracks, a transverse crack with straight crack front is one of conventional crack types.

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Hence, many researchers focused on modeling the flexibility and analyzing the nonlinear dynamic behavior of rotors associated to this crack type.

On 1870s, Papadopoulos and Dimaragonas investigated longitudinal and bending vibrations of a cracked shaft through the simulation of flexibility matrix which comes from the crack [2,3]. They divided cracked section of the shaft into strip elements, and computed the flexibility matrix of the crack by applying governing stress intensity factor equations and fracture mechanics approach. This method has also been used to simulate bending and torsional vibrations [4]. Sinou studied the crack behavior by reduction of second moment of area (ΔI) of the lumped element at the crack location [5]. He also presented a truncated Fourier series to study the breathing phenomenon. The transverse crack by the same strip elements was also simulated by Chen and he investigated the stability of the rotor influenced by the crack and asymmetric supports [6]. Jun modeled additional slope generated by crack subjected to bending moment by strip elements method and fracture mechanics. He applied the transfer matrix method to study a conventional rotor composed of multiple shafts, disks and cracks [8,9]. Darp simulated complete flexibility matrix of a transverse crack with a straight front by mentioned method [10]. Various articles investigated this crack type by the above method [7,11–13]. Also, numerous scholars such as Sinou and Lees [5] studied this crack shape by applying the method of reduction of second moment of area [14–17]. Another kind of transverse crack shape is elliptical crack front. Shin and Cai [18] presented the modeling of this crack type in order to compute stress intensity factors. Local flexibility due to this crack type was investigated by the results of [18] and the same concept of straight front (strip elements) [19,20].

Among published manuscripts, limited articles have investigated circumferential transverse crack and dynamic behavior of cracked rotors in the presence of this crack type. Such a crack was first introduced by Dimarogonas and Massouros [21]. They comprehensively investigated torsional vibration of a shaft in the presence of a symmetric circumferential crack. For this purpose, torsional flexibility coefficient of crack was computed by applying the analytical method. Their results showed that the change in dynamic response due to crack is high enough to detect the crack and estimate its location or magnitude [21]. Vania et al. investigated a circumferentially cracked steam turbine and calculated order $1X$ and $2X$ of cracked section moment of inertia through an iterative procedure. Their study showed that circumferential crack causes a minor change in shaft super-synchronous vibration and their findings were validated by experimental data [22]. In another study, they investigated the effect of the thermal transient of the steam turbine during run-up and coast-down on rotor vibration. They found that thermal expansion of the turbine during run-up causes crack closure and the reduction of turbine vibration due to crack [23]. Vania and Pennacchi also presented a method to identify circumferential crack called shape frequency plot. Indeed, they obtained natural frequency of annular cracked steam turbine in static condition through experimental modal analysis in every step of circumferential direction and illustrated its variation in this direction in a polar plot. This diagram is an efficient method to identify this kind of crack [24]. Torsional vibration of a shaft in the presence of a symmetric circumferential crack was studied by the variational method too [25]. Yngvesson presented an expression to compute stress intensity factors of an asymmetric circumferential crack subjected to axial and torsional loadings [26]. This work compared the results of presented equations to the results of finite element simulations in small magnitude of eccentricity. This comparison does not show a good compatibility. So, it can be concluded that the presented equations do not have enough accuracy for computing local flexibility of crack. Vaziri simulated this kind of crack using the same method of Dimarogonas [2] and investigated the effect of the plastic region in crack front and friction of crack surfaces to analyze the reduction of the torsional vibration magnitude of the shaft [27].

Although extensive works have conducted in this area, most of the previous papers have just reported symmetric circumferential crack subjected to torsional loading, up to now and there is no analytical method to compute additional flexibility due to this crack type like transverse crack with straight or elliptical crack front. In this paper, local flexibility of an asymmetric circumferential crack subjected to bending, torsional and axial loadings is investigated. Also, the effects of unilateral contact between crack surfaces on bending and longitudinal stiffness are studied. Indeed, this paper aims to describe the derivation of an analytical method to model a lumped asymmetric circumferential cracked beam with contact condition. This model is applicable for modeling cracked rotor and analyzing breathing phenomenon. Result of presented model is compared to numerical results of 3D finite element method (FEM).

2. Modeling of circumferential crack

Circumferential crack is one of the crack shapes that have been seen in the shaft of turbo generators. A parametric modeling of this crack type has been carried out in [29,30] in order to survey its effect on the shaft vibrational behavior. However, there is no analytical formula to model the breathing phenomenon and additional deformations due to this crack type, like transverse crack with straight crack front. Fig. 1 shows an asymmetric circumferential crack in which parameters e, c, and b are eccentricity, radius of un-cracked area and the shaft radius, respectively. If parameter e is equal to zero, this crack becomes the same symmetric circumferential crack and additional slop and longitudinal displacements due to crack can be calculated by using a strain energy density function and the Paris law as follows:

$$J = \frac{1-\nu^2}{E}\left[\left(\sum_{i=1}^{6} K_{I_i}\right)^2 + \left(\sum_{i=1}^{6} K_{II_i}\right)^2\right] + \frac{1+\nu}{E}\left(\sum_{i=1}^{6} K_{III_i}\right)^2 \quad (1)$$