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# Hybrid Evidence Theory-based Finite Element/Statistical Energy Analysis method for mid-frequency analysis of built-up systems with epistemic uncertainties

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ABSTRACT

Considering the epistemic uncertainties within the hybrid Finite Element/Statistical Energy Analysis (FE/SEA) model when it is used for the response analysis of built-up systems in the mid-frequency range, the hybrid Evidence Theory-based Finite Element/Statistical Energy Analysis (ETFE/SEA) model is established by introducing the evidence theory. Based on the hybrid ETFE/SEA model and the sub-interval perturbation technique, the hybrid Sub-interval Perturbation and Evidence Theory-based Finite Element/Statistical Energy Analysis (SIP-ETFE/SEA) approach is proposed. In the hybrid ETFE/SEA model, the uncertainty in the SEA subsystem is modeled by a non-parametric ensemble, while the uncertainty in the FE subsystem is described by the focal element and basic probability assignment (BPA), and dealt with evidence theory. Within the hybrid SIP-ETFE/SEA approach, the mid-frequency response of interest, such as the ensemble average of the energy response and the cross-spectrum response, is calculated analytically by using the conventional hybrid FE/SEA method. Inspired by the probability theory, the intervals of the mean value, variance and cumulative distribution are used to describe the distribution characteristics of mid-frequency responses of built-up systems with epistemic uncertainties. In order to alleviate the computational burdens for the extreme value analysis, the sub-interval perturbation technique based on the first-order Taylor series expansion is used in ETFE/SEA model to acquire the lower and upper bounds of the mid-frequency responses over each focal element. Three numerical examples are given to illustrate the feasibility and effectiveness of the proposed method.



## 1. Introduction

In the past decades, there has been an increasing interest in predicting the dynamic response of systems with uncertain parameters. Before perform an uncertain analysis, identifying the types of various uncertainties is important, sometimes even imperative. Uncertainties can be classified as two distinct types [1]: aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty, also referred as objective or stochastic uncertainties, which is always modeled by using the probability theory [2]. Conversely, the epistemic uncertainty is a subjective uncertainty that is due to the lack of knowledge in quantifying the uncertain system. This type of uncertainty is reducible in the sense that the uncertainty level will be decreased

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when more information is collected. To handle the epistemic uncertainty, different kinds of theories have been developed in the last two decades, such as the convex models [3], the fuzzy sets [4,5], the possibility theory [6] and the evidence theory [7,8]. Among the mentioned approaches above, the evidence theory has a much more flexible framework than other modeling techniques. Under different cases, it can provide equivalent formulations to classical probability theory and convex models, respectively. In addition, the evidence theory can deal with conflicting information from experts. Due to these advantages, the evidence theory has been widely applied to solve the uncertain engineering problems recently [8–10]. But so far, the evidence theory based numerical methods are mainly used for the structural static response analysis, dynamic response analysis and reliability analysis in the low-frequency range [9–12].

The analysis of vibration or vibro-acoustic problems usually involves a broad frequency range due to the broadband nature of the external excitations. Theoretically, the numerical technique would be applicable over the whole frequency range of interest when it is used for modeling the vibration system. But in practice specific methods are only applicable in limited frequency range. Element-based methods like the Finite Element Method (FEM) [13] have been widely used in the low frequency range, where the wavelengths are short relative to the dimensions of the systems. However, because the computational effort of FEM typically increases exponentially with the increasing of frequency, it is unacceptable to analyze the high-frequency system by FEM. In the high frequency range, the Statistical Energy analysis (SEA) [14,15] is commonly employed. Based on the principle of conservation of energy, SEA yields a good prediction for the statistical behavior of the system with low computational time. The basic assumptions of SEA is that all subsystems are sufficiently random, which restricts its use to the high-frequency range.

In the mid-frequency range, some built-up systems may consist of both short-wavelength and long-wavelength components [16,17]. For this type of mid-frequency vibration problem, neither the low-frequency method such as the FE method, nor the high-frequency method such as the SEA, can be suitable: the FE method requires too many degrees of freedom to capture the dynamic behavior of the short-wavelength components, while the underlying assumption of SEA may not be met for the long-wavelength components. To solve this mid-frequency vibration problem, Langley and co-workers have recently proposed a hybrid FE/SEA approach[17] in which some components of the system are considered to be deterministic and modeled via FE, while other components are assumed to be highly random and modeled by SEA. Based on the diffuse field reciprocity relation between the FE components and the SEA components, the hybrid FE/SEA approach allows us to predict the main mid-frequency vibration responses such as the ensemble mean and variance of the energy response and the cross-spectrum response. The hybrid FE/SEA approach has successfully coupled the determined method to the statistical method, and is widely used for the response analysis of structural-acoustic systems and vibration systems in a mid-frequency excited environment [18–20].

Within the hybrid FE-SEA framework, the uncertainty of the SEA components is modeled as non-parametric in the sense that the SEA components are assumed to be highly random, so that the natural frequencies and mode shapes conform to certain universal statistical distributions regarding the underlying uncertainties; while the properties of the FE components are assumed to be deterministic. However, due to the manufacturing imperfections and aggressive environmental factors in practice, uncertainties associated with the material properties, geometric dimensions, applied loads and other parameters of the FE components are unavoidable. These parametric uncertainties would have an enormous impact on the mid-frequency responses of the vibration system. Up to now, the research on the FE/SEA model with parametric uncertainties is still in its preliminary stage. Recently, Cicirello and Langley introduced parametric uncertainty into the FE components of the hybrid FE/SEA model by using the interval analysis and the probability approach [21,22]. The probability approach is considered as the most valuable method for the uncertain problems with aleatory uncertainties. Unfortunately, in the early stage of design, the parameters of the FE subsystem are usually involved with epistemic uncertainties due to the lack of data or knowledge. In this case, both the probability approach and the interval analysis have their own disadvantages: within the probability approach, some suitable assumptions for the probability distributions of uncertain parameters have to be made; within the interval analysis, the information about the probability distributions of the uncertain parameters is missing. The corresponding uncertain responses obtained from these two methods may be unreliable or incorrect. Thus, both the probability theory based and the interval analysis based hybrid FE/SEA method are not appropriate for the mid-frequency vibration problems with epistemic uncertainties.

The evidence theory has a strong ability to handle both aleatory and epistemic uncertainties without any baseless assumptions. Besides, it can deal with the conflicting information from experts. In this paper, the evidence theory is introduced into the hybrid FE-SEA framework, and the hybrid Sub-interval Perturbation and Evidence Theory-based Finite Element/Statistical Energy Analysis (SIP-ETFE/SEA) method is proposed to analyze the frequency response of built-up systems with epistemic uncertainties in the mid-frequency excitation range. In the proposed method, the uncertain parameters of the FE components are modeled as evidence variables. By employing the evidence theory to propagate the evidence variables within the hybrid FE/SEA model, the distribution characteristics of the mid-frequency responses of built-up systems can be obtained. In order to alleviate the computational burdens, the sub-interval perturbation method is used to acquire the approximate mid-frequency response bounds for each focal element. Three numerical examples on the built-up systems with epistemic uncertainties are presented to verify the effectiveness of the proposed method.

The remainder of this paper is organized as follows. The main concept of evidence theory is introduced in Section 2. In Section 3, the theoretical principle of the hybrid FE/SEA method is briefly summarized. In Section 4, the hybrid SIP-ETFE/SEA method to predict the mid-frequency responses of built-up systems with epistemic uncertainties is proposed. Two numerical examples are investigated in Section 5 and some conclusions are given in Section 6.