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A continuum membrane model for small deformations of a spider orb-web

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ABSTRACT

In this paper we propose a continuum membrane model for the infinitesimal deformation of a spider web. The model is derived in the simple context of axially-symmetric webs formed by radial threads connected with circumferential threads belonging to concentric circles. Under suitable assumption on the tensile pre-stress acting in the referential configuration, the out-of-plane static equilibrium and the free transverse and in-plane vibration of a supported circular orb-web are studied in detail. The accuracy of the model in describing a discrete spider web is numerically investigated.

1. Introduction

The spider orb-web is a complex biological-mechanical system that has attracted increasing interest in the scientific literature in the last four decades. This web is a natural, lightweight, elegant structure with an extreme strength to weight ratio that is rarely observed among other structures, either natural or manmade. Its primary functions are for catching prey and sensory information, and the study of the mechanisms guiding the spider in prey capture and gathering information through web vibration has been - and actually is - one of the main objectives of the research in this field. Interspecific and intraspecific variations in the structure of orb-webs is widely documented [1–3]. Moreover, individual spiders adapt the characteristics of the web to specific conditions such as nutritional status, spider size, presence of parasites or predators, type of prey available, weather conditions, or nutritional status [4]. Integrating both biological and biomechanical approaches can help to uncover how web architecture suits for vibratory sensing and prey catching under different circumstances, and to identify selective pressures that have guided their evolution. Basic questions posed by the researchers mainly concern: (i) how spiders might discriminate between the large set of web-borne vibrations and, particularly, between prey-produced signals and irrelevant vibrations such as those generated by wind; and (ii) how exactly the spider adjusts the web mechanics to the environmental conditions where the web is built, such as, the pre-stress tensile forces to be assigned at specific locations of the web, in order to improve its ability to prey capture. The answers to both questions fall on the understanding of the different physiological/behavioral processes. The use of modelling approaches could help to uncover the consequences behind the biological adaptations and evolutionary success of spiders.

In a series of papers published in the 80's, Master and co-authors studied, both experimentally and analytically, the vibration transmission through the web. Signal amplitude was very low and simplified linear dynamic models were used for the

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interpretation of the experiments. In [5,6] it was assumed that the radial strands are the most important vibration-conducting elements of the web. Accordingly, the types of vibrations propagating through the web were classified into longitudinal, in-plane lateral and out-of-plane transverse. The conclusion was that, in an empty web, longitudinal vibration is almost not attenuated and, then, it plays a dominant role in the spider's choice of the path to reach the trapped prey. From the experiments emerged that the presence of a spider at the hub of the web induces a significant attenuation of the longitudinal vibration, and Masters [7] concluded that for prey detection and recognition all types of vibrations may be equally important. The first analytical estimates of resonant frequencies of spider orb-webs are due to Frohlich and Buskirk [8]. The authors used simple mechanical models with lumped masses connected to crossing stretched threads. Landolfa and Barth [9] used in their experiments transmission of natural and artificial vibrations in webs to determine how spiders discriminate and locate the stimuli. Using a multidisciplinary approach, Mortimer et al. [10] studied vibrations in spider silks in comparison with other materials, identifying evolutionary trade-offs between mechanical and signalling functions. All the above analyses were strongly based on the one-dimensional character of the wave propagation through the web, even if it was more or less explicitly recognized in the literature the need of developing a vibration analysis of the web described as a two-dimensional sheet of threads.

In parallel to investigations on signal transmissions on the web, there has been an enormous increasing of research on bio-mechanical aspects of spider orb-webs [2]. Vollrath et al. [11] investigated on the effect of environmental and physiological variables, such as web support, wind, temperature, humidity and silk supply, on web geometry [12]. Wirth and Barth [13] presented the first in situ measurement of the pre-stress forces in individual threads of intact spider webs, and provided arguments supporting the hypothesis of tension control by the spider. Gosline et al. [14] highlighted the influence of the dissipative behavior of the silk in the energy absorption capacity of the web, as compared to other constitutive behaviors. The effect of increases in thread tension on prey-detection efficiency was examined by Nakata [15]. Concerning the progress on the study of mechanical characterization of the spider silk, the experiments carried out by Ko and Jovicic [16] showed, among other aspects, that spider silk has toughness properties significantly higher than that of the strongest man-made fibers in tension, see also Harmer et al. [4]. The experiments and simulations performed by Cranford et al. [17] allowed to identify the nonlinear response of silk threads, and confirmed the superior resistance to structural defects (i.e., broken threads) in the spider web compared to other linear elastic or elasto-plastic materials. Hesselberg and Vollrath [18] investigated the mechanical behavior of the non-sticky permanent spiral in *Nephila* webs and, in particular, on the stress assigned by the spider to this spiral during web building.

Upon reviewing the literature it emerges that several issues of the two main questions (i) and (ii) posed at the beginning of this Introduction are still open. One of the reasons is probably connected with the lack of analytical/numerical studies on the mechanical behavior of the spider web as whole two-dimensional structure. The recent development of highly sophisticated numerical models of spider webs has partially overcome this limitation. Finite element analysis and numerical methods have been identified and used by different authors as a valuable tool to integrate detailed data on web structure and silk properties, allowing to understand how silk biomechanics and web architectures interacted to influence spider web evolution along different structural pathways [19,16,20,23,4,21,22,17,26–28]. For example, the effect of damage on the static and dynamic response of a web was numerically investigated by Ko and Jovicic [16] and Alam et al. [23]; an analysis of high performance spider silk was presented by Cranford et al. [17], Pugno et al. [24], Qin et al. [25]; the role of aerodynamic drag in the dissipation of prey's energy and in reducing deterioration of the orb web was considered by Zaera et al. [26]; the key effect of the secondary frame in avoiding excessive stiffness in radials was analyzed by Soler and Zaera [27]; links between silk material properties and propagation of vibrations within webs were studied in Mortimer et al. [28]. These models offer a remarkable versatility and accuracy in reproducing the response of the web under wind loads, prey impacts or vibratory excitation. However, theoretical models often permit a deeper insight in the physical phenomena through the analysis of the underlying mathematical structure of the governing equations, which can be also written in nondimensional form to identify the most relevant parameters that rule the response of the web. The first two-dimensional model of spider web was proposed by Aoyanagi and Okumura [29,30]. The model consists of radial and circumferential threads, and each thread is described as a stretched spring subject to pre-stress tensile force in the referential configuration. The model was used to determine the pre-stress state in an intact axially-symmetric web, and in a web damaged by removing some circumferential threads. For the intact web, an infinitesimal homogeneous radial deformation was assigned from the unstressed state, and an analytical solution was provided. An approximate solution was proposed in case of damaged web. Numerical simulations showed that when radial threads are sufficiently strong compared to the circumferential threads, the damaged web is free of stress concentration, in contrast with what occurs in common materials. The model by Aoyanagi and Okumura was purely static, and its possible use for the study of either in-plane or out-of-plane response was not investigated, not even under the hypothesis of small deformations of the web.

In this paper we present a continuous mechanical model for small deformations of a spider orb-web. The actual discrete web, formed by a finite number of radial and circumferential threads, is approximated by a continuous elastic membrane on the assumption that the spacing between threads is small enough. The continuous membrane has a specific fibrous structure which is inherited from the original discrete web, and it is subject to tensile pre-stress in the referential configuration. Although the model can be adapted to reproduce general geometries, for simplicity here we restrict the attention to circular-shaped webs in which the circumferential threads belong to concentric circles. Furthermore, we study in detail the static and dynamic response of the web supported at the external boundary.