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Resonator-based detection in nanorods

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ABSTRACT

In this paper the axial vibrational behavior of nanorods with an attached point-mass is studied, using the modified strain energy theory. The natural frequencies of the nanorod with the concentrated mass are obtained for different boundary conditions. The effects of the concentrated mass intensity, mass location, as well as the value of scale parameters have been analysed. For the case of small intensity of the concentrated mass, the natural frequencies of the nanorod can be estimated using a first order perturbative solution. These approximate results are compared with those corresponding to the exact solution. For this case, from the properties of the eigenvalue perturbative theory, the identification of single point mass in uniform nanorods (mass intensity and position) is addressed. The results obtained encourage the use of axial vibrations of nanorods as a very precise sensing technique.

1. Introduction

Nowadays, the scientific community interest is attracted by the use of nanostructures (carbon nanotubes, *CNTs*, graphene sheets, *GSs*, and nanowires) as nano-sensors. The reason is connected with the promising features regarding a wide range of applications such as gas detection, early disease detection, gene mutation detection, DNA sequencing. In this respect, several reviews have been recently published showing the different capabilities of the nanostructures [1–3]. According to Khanna [4], nano-sensors can be classified into six groups: mechanical, electrical, optical, magnetic, chemical, and thermal.

In this research we are interested in mechanical nanoresonator sensors and, in particular, in vibration based-methods as identification techniques. The sensing principle for this class of nanoresonators is based on the measurement of the variations of the resonant frequencies caused by (unknown) additional masses located on the initial system. The conventional detection principle assumes that the mass perturbation, caused by attachments of foreign atoms or molecules, chemical/molecular adsorption, the presence of virus particles or protein-protein and protein-DNA interactions, can be described as Dirac-delta point masses, having unknown intensities and locations, superimposed to the given mass density of the nanoresonator. We refer to [1,5] for more sophisticated mechanical models in which a simultaneous perturbation of the stiffness properties coupled with the mass increase is also considered in the analysis. Our main goals in the present research are: (i) to derive a continuum mechanical model able to describe the axial vibration of a nanoresonator with a single additional point mass, and (ii) to develop a method for the identification of the point mass from minimal eigenfrequency data. In particular, we shall consider the inverse problem in which the added mass is *small* with respect to the total mass of the nanosensor.

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It is well known that the size effects are significant regarding the mechanical behavior of the nanostructures which composes the nanoresonators. The large computational effort required for the Molecular Dynamics techniques (*MD*), see, among others [6–9], encourages the exploration of other possibilities, such as generalized continuum mechanics approaches given that classical continuum mechanics cannot predict the size effect, due to its scale-free character.

Among the generalized continuum theories, we cite here three main groups: Cosserat micropolar elasticity [10], the strain gradient elasticity of Mindlin [11,12], and the nonlocal continuum mechanics initiated by Eringen and coworkers [13,14], and formulated originally in integral form.

From the early integral nonlocal theory, Eringen [15] introduced a differential constitutive theory showing that, for a specific class of kernel functions, the non-local integral constitutive equation can be transformed into a differential form, much easier to manage than the integral model. From the pioneer work of Peddieson et al. [16], this differential version of the Eringen nonlocal model has been widely used to address the mechanical behavior (static and dynamic) of nanostructures. The list of papers related with these applications is extremely long to be reported here. The interested reader can see the very recent review by Rafii-Tabar et al. [17].

Several authors used the Eringen elasticity theory to asses the vibrational behavior of beams and rods with attached masses. Thus, Elthaer et al. [18], Murmu et al. [19], and Li et al. [20] applied this theory to obtain the shift of the natural frequency of bending vibrations of nanobeams carrying attached mass. Moreover, Murmu et al. [19] and Li et al. [20] provided identification formulas from the approximated expressions of the frequency shift. However, the cases studied in the above papers are rather specific and correspond to a nano-cantilever with a mass attached at the tip or a distributed mass through a certain length from the tip [19], while three configurations, corresponding to a cantilever beam with a mass attached at the tip, simply supported, and bi-clamped beams with the mass attached at the mid-section, are analysed in [20].

Regarding the use of the Eringen elasticity theory applied to *CNTs* with a single attached mass vibrating in axial direction, it is worth to note the work by Aydogdu and Filiz [21] who analyzed the frequencies of axially vibrating *CNTs* (clamped-clamped and clamped-free) with a single attached mass located at different positions.

Li et al. [22] studied the natural frequencies of an axially vibrating nanorod with an elastically restrained end by a nanospring (depending of the stiffness of the nanospring this end could be considered clamped or free), and with an attached mass at the other end considered free. In this analysis, the Love hypotheses are considered (i.e. the inertial effects of radial motion have been taken into account).

Nevertheless, several authors have pointed out some inconsistencies arising from the Eringen differential model when it is applied to the static behavior of bars in tension [23], static bending behavior of a Euler-Bernoulli beams [16,24–27] or flexural vibrations of a cantilever beam [28].

Recently, Fernández-Sáez et al. [29] and Romano et al. [30] give some new insights on the origin of these inconsistencies. Therefore, a more suitable approach to describe the mechanical behavior of the nanostructures is needed.

The modified strain gradient elasticity theory was proposed by Lam et al. [31] based on previous developments by Mindlin [12] and Fleck and Hutchinson [32]. This approach needs new additional equilibrium equations to govern the behavior of higher-order stresses, and contain only three non-classical constants for isotropic linear elastic materials. Some papers can be cited to illustrate (the list is not exhaustive) the use of this theory to model the mechanical behavior of 1D simple nanostructures (beams and rods). Thus, using this approach the static and dynamic bending behavior of Euler-Bernoulli beams [33] and Timoshenko beams has been studied [34]. Akgoz and Civalek [35,36] obtained analytical solutions for the buckling problem of axially loaded nano-sized beams. The free torsional vibrations of microbars have been analyzed in [37,38]. Akgoz and Civalek also studied the longitudinal vibrations of homogeneous [39] and nonhomogeneous (functionally graded material) [40] microbars using the simple rod theory, while Guven [41] analyzed the propagation of longitudinal stress waves based on Love-Bishop hypothesis, i.e. considering the lateral deformation and the shear strain effects.

To our knowledge, there is no theoretical investigation on the axial vibrations of nanorods with attached concentrated mass when the modified strain gradient elasticity theory of Lam et al. [31] is used as constitutive model. This analysis is relevant regarding the nanosensor applications of this kind of structures.

Regarding the experimental determination of frequencies in axially vibrating nanorods, some papers can be found in the literature (see for instance [42–44]). However, to the authors knowledge, no experimental works dealing with axially vibrating nanorods with attached masses have been published.

In this paper we analyze the axial vibrational behavior of a nanorod carrying a concentrated mass through its span and subjected to different boundary conditions. The mechanical behavior of the nanorod is modeled using the modified strain gradient theory proposed by Lam et al. [31]. The effects of the mass intensity, location as well as the value of scale parameter have been analyzed. For the case of small intensity of the concentrated mass, a first order perturbative technique is used to estimate the natural frequencies of the nanorod. The approximate results are compared with those corresponding to the exact solution. Basing on the explicit expression of the first-order eigenfrequency change induced by the point mass, we are able to formulate and solve the inverse problem consisting in the identification of the location and intensity of the point mass in a uniform nanorod from minimal eigenfrequency data. In particular, for nanorods under a specified set of end conditions, the method gives closed-form expressions of both the location and the intensity of the point mass in terms of a suitable pair of eigenfrequencies of the nanorod.

The paper is organized as follows. The mechanical model of the nanorod under longitudinal free vibration with and without point mass is briefly recalled in Section 2. Section 3 is devoted to the illustration of the perturbation effects of the small added mass on the eigenvalues of the nanorod. The inverse problem of identifying the position and the intensity of the small