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Construction of a statistical model for the dynamics of a base-driven stick-slip oscillator

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A B S T R A C T

In this paper the dynamics of a dry-friction oscillator driven by a stochastic base motion has been analyzed. The system consists of a simple oscillator (mass-spring) moving on a base with a rough surface. This roughness induces a dry-frictional force between the mass and the base which is modeled as a Coulomb friction. It is considered that the base has an imposed stochastic bang-bang motion which excites the system in a stochastic way. The non-smooth behavior of the dry-frictional force associated with the non-smooth stochastic base motion induces in the system stochastic stick-slip oscillations. A statistical model is constructed for the stick-slip dynamics of the system. The objective is to characterize, from a statistical view point, the response of the dry-friction oscillator composed by a sequence of stick and slip-modes. Defined a time interval for analysis, some of the variables which appear in this statistical model are the number of time intervals in which stick and slip occur, the instants at which they begin and their duration. These variables are modeled as stochastic objects. Statistics of them, as mean, variance and entropy, and histograms, are computed by the integration of the dynamics equations of the system using independent samples of the base movement generated with the Monte Carlo method.

1. Introduction

The analysis of a stick-slip dynamics caused by dry-friction is not a new subject. The literature dealing with the problem is vast (see for instance [1–4]) and, reflects the economic interest in understanding the dynamic behavior of this type of non-smooth vibration [5–7]. Dry-friction appears in several situations, as in drilling process and in mechanical gear systems [8–10]. It could be the source of dynamic instability, noise, and reduction of performance [11,12]. In drilling, stick-slip oscillations happen due to the dry-friction force that exists between the bit and the rock. This friction force could be big enough to stuck the bit during some time intervals. As a constant velocity is imposed at the top of the drillstring and during the stick the bit at the bottom does not move, the drillstring is twisted. It accumulates energy in terms of torsion up to the instant that it is suddenly released. This phenomenon generates torsional vibrations and instabilities in the system dynamics. If not controlled, it can be harmful to the drilling process and causes waste of energy. Furthermore, when the bit is stuck, there is no penetration. Due to all these reasons, in this case the stick-slip oscillations are undesirable and should be avoided.

Usually, stick-slip oscillations appear in mechanical systems in which uncertainties play an important role. For example in drilling, some of sources of uncertainties are the bit-rock interaction, the presence of impacts, and fluid-structure interaction [13,14]. In gears, randomness arise from manufacturing, assembly errors, and random load. Beyond these sources of

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uncertainties, the dry friction force itself presents an inherent random behavior [15]. The influence of ambient conditions in the properties of contact surfaces [16,17] and the dependency on the relative velocity of the bodies in contact turn the dry friction force uncertain. Because of this, a stochastic approach is the ideal way to address the problem of dry-friction [18–20]. The source of uncertainties of the dry-friction oscillator analyzed in this paper is the base motion. It is considered that the base has an imposed stochastic bang-bang motion which excites the system in a stochastic way.

In [21] a similar dry-friction oscillator to the one found in this paper was analyzed. In this first work the duration of the stick-mode has been analyzed from a deterministic and also from a stochastic viewpoint. The focus was to understand what are the parameters that most affect stick duration. The paper [21] was one of the rare works that appear in the literature dealing with the duration of stick-mode, despite the economic interest of the subject.

Here a more complete analysis of the response of a stick-slip oscillator is performed. The objective here is to construct a statistical model for the response of a stick-slip oscillator. In this statistical model, some of the variables of interest are the number of time intervals in which stick and slip occur, the instants at which they begin, and their durations. The system response presents a sequence of stick and slip-modes and we are interested in the statistical characterization of these sequences. We believe that the construction of a statistical model of the response is a novelty in the literature since no references dealing with it were found after an extensive literature review.

In the statistical model, the variables of interest are modeled as stochastic objects. By numerical simulations, statistics of them, as moments, histograms, and entropy were estimated and analyzed. The first moment allowed a characterization the mean response of the stick-slip oscillator. With this mean response, we know in average how many sticks will happen. We know also, in average, when sticks will start and how long they will last. We believe that the theoretical knowledge on these means can help in the development of robust control techniques to avoid, or diminish, stick-mode. The idea is that predicting the instant in which a stick will start, the controller can anticipate an action on the system that will avoid the beginning of stick or at least reduce its duration. One possible application is in the robust optimization of the drilling process.

Moreover, the histograms allowed a deeper analysis. They gave us much more information than simply the means and moments. With them, we got information about the probability distributions of the stochastic objects, since for a large number of realizations, a histogram approaches the probability density function. Therefore, analyzing the histograms we got information about the probabilistic model of the system response and we could better characterize the stick-slip process, i.e., the sequence of stick and slip-modes which compose the system response. A question of interest is to determine if the durations of sticks are independent and identically distributed random variables. The same questions can be answered to the durations of slips and instants at which sticks and slips start. It is important to remark that such analysis is new in the literature.

Besides this probabilistic characterization of the stick-slip process, we believe that these histograms represent another tool that could be used the development of robust control techniques to avoid or diminish stick-mode. In a technique with a Bayesian approach, the histograms would behave as a prior distribution of the system response.

This paper is organized as follows. Section 2 describes the most simple stick-slip oscillator. The construction of the probabilistic model of the uncertain base motion is given in Section 3. The construction of the statistical model of the system response by the definition of the stochastic objects which are the variables of interest in a stick-slip problem is given in Section 4. Statistics, as moments, entropy, and histograms of the number of time intervals in which stick occur, the instants at which they begin, and their duration are presented in Section 5.

2. Dynamics of the stick-slip oscillator

The system analyzed is composed by a simple oscillator (mass-spring) moving horizontally, in one-dimension, on a rough surface, called table, or base, as shown in Fig. 1. The roughness induces a dry-frictional force between the mass and the base which is modeled as a Coulomb friction.

Due to this friction model, the resulting motion of the mass can be characterized in two qualitatively different states of motion, called modes: stick-mode (in which mass and base have the same velocity during an open time interval) and slip-mode, in which mass and base have different velocities [22–26]. The position of the mass over the base is represented by x and its equation of motion is

$$m\,\ddot{x}(t) + k\,x(t) = f(t), \tag{1}$$

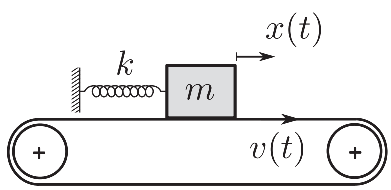

Fig. 1. Stick-slip oscillator: mass-spring system moving on a rough table.