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Analytical and experimental characterization of nonlinear coned disk springs with focus on edge friction contribution to force-deflection hysteresis

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ABSTRACT

Even though coned disk springs are widely used given their highly customizable design features, the literature on both analytical and experimental characterizations is sparse. This article overcomes the void by focusing on midrange displacements for a commercially available coned disk springs with square edges. A refined single disk spring model, capable of having asymmetric friction conditions imposed on the disk spring edges, is developed to predict the load-deflection and quasi-static hysteresis characteristics while relaxing prior limiting assumptions. A new quasi-static experiment is then proposed to measure the nonlinear load-deflection characteristics under four principal interfacial edge configurations — two symmetric and two asymmetric. In particular, the asymmetric friction conditions afford the opportunity to assess the validity of the long standing theory concerning the assumed location of the disk spring cross section rotation point. The imposition of asymmetric friction conditions on the disk spring also allows for the direct quantification of the contribution of asymmetric friction conditions on the disk spring contact edges. New analytical and experimental studies show that there are significant edge friction contributions to the hysteresis exhibited by the disk spring. Finally, stiffness parameters from both theory and experiment are briefly evaluated over midrange displacements.

1. Introduction

Coned disk springs, sometimes known as Belleville washers, belong to a family of diaphragm springs which assume several forms, such as wave washers and finger springs [1–4]. Coned disk springs are highly customizable for the desired application because of their unique stiffness and damping properties. The most common application for which disk springs are used is the application of preload; however, it has recently become apparent that it may be possible to leverage the disk spring's natural quasi-zero stiffness regime for the purpose of vibration isolation [1,2,5–8]. Overall, the literature concerning disk springs is relatively limited despite their utilization for over a century [3,4,9–26]. In particular, the experimental data from disk springs is sparse and most prior experimental studies mirror the simplifications used in analyses on which they are based. Therefore, the primary goals of this article are to develop a refined load-deflection model, propose a new disk spring experiment which is independent of preconceived analytical assumptions, and determine the nonlinear load-deflection relationships. The scope of this article is limited to commercially available coned disk springs with square edges as shown

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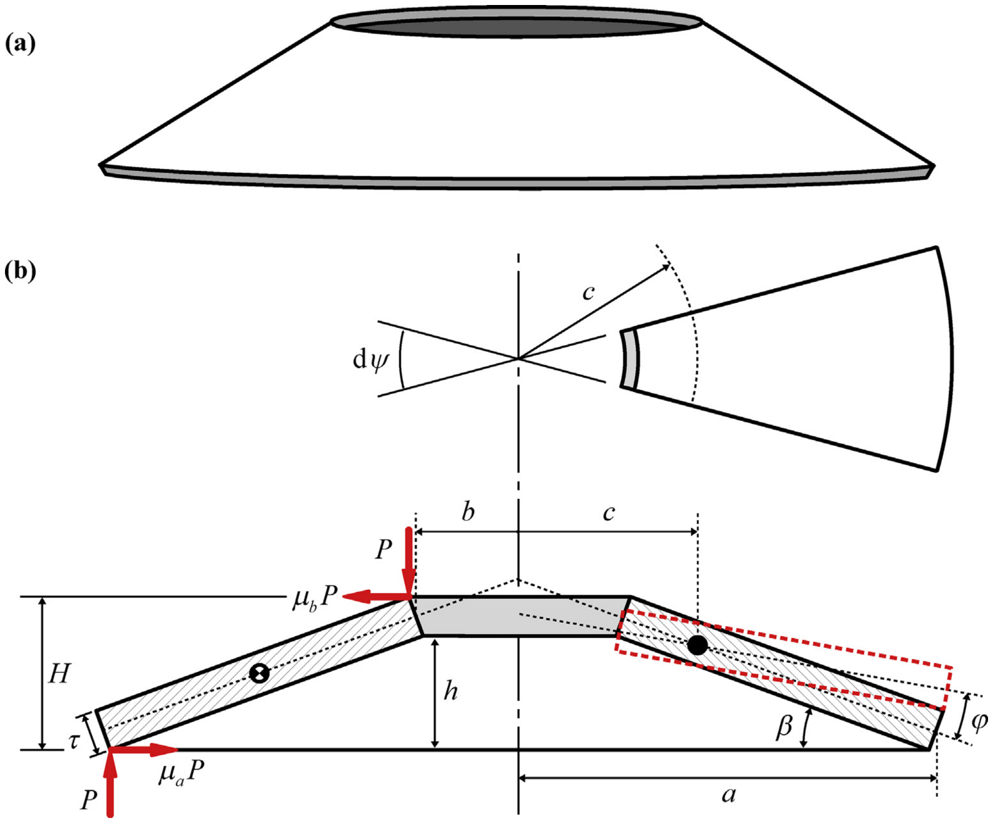

Fig. 1. Example case. (a) Coned disk spring with square edges; (b) top and cross section views of a coned disk spring with geometric parameters and loads: - - - cross section deformed position, → edge contact force. Symbols are defined in the text and in Appendix A.

in Fig. 1a; however, the methodology could be extended to diaphragm spring in general within the specified displacement range. The displacement range of interest is restricted to the midrange of their typical working displacements such that the resulting load-deflection results may be examined without influence from either initial nonlinearity due to asymmetry in the system or the characteristic stopper effect seen in load-deflection results.

2. Literature review

A method of analytically finding the load-deflection relationship was first proposed by Timoshenko as a textbook example [22] and his assumptions have permeated most of the subsequent disk spring analysis and experimentation. For instance, the following assumptions are still employed today: (a) the cross section's angular displacement is small; (b) there is no distortion of the cross section as it rotates about its neutral point during the loading process; (c) the forces at the disk spring boundaries are evenly and concentrically distributed; and (d) the radial stresses are negligible [9,21,22]. Almen and Laszlo, in their seminal paper of 1934 [9], expanded and generalized Timoshenko's example by including key aspects of disk spring kinematics which had been previously neglected; it remains the current *de facto* technical standard [9,18]. Major consequences of these assumptions are that the disk spring remains conical throughout the deformation process, and the boundary conditions at the upper and lower edges would not be satisfied [23]. Curti and Orlando [10] treated the coned disk spring as an annular plate, and linked the radial and tangential stresses by an equilibrium equation. Since Almen and Laszlo [9] erroneously neglected radial strain instead of radial stress, Curti *et al.* [13] correctly applied this assumption thereby allowing the natural conic geometry of the disk spring to be maintained. The formulation presented in Curti *et al.* [13] results in a load-deflection relationship that differs by a factor of $(1 - \nu^2)$, where ν is the Poisson's ratio, from the denominator from the original Almen-Laszlo result.

Although Almen and Laszlo [9] pointed out that the friction forces within a disk spring stack could be varied via the stacking configuration, the presence of friction during the loading process has been largely neglected in most modeling efforts. Curti and Montanini [17] expanded the relationship presented in Curti *et al.* [13] by the addition of a Coulomb friction term to account for the friction forces at the outer and inner contacting edges of a single disk spring; it is predicated on the theoretical imprecision that the cross section rotates about its geometric center. This assumption implies that the outer and inner contact edges would translate the same distance in opposite directions during the loading process, and is of no real