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Free response approach in a parametric system

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ABSTRACT

In this study, a new approach to predict the free response in a parametric system is investigated. It is proposed in the special form of a trigonometric series with an exponentially decaying function of time, based on the concept of frequency splitting. By applying harmonic balance, the parametric vibration equation is transformed into an infinite set of homogeneous linear equations, from which the principal oscillation frequency can be computed, and all coefficients of harmonic components can be obtained. With initial conditions, arbitrary constants in a general solution can be determined. To analyze the computational accuracy and consistency, an approach error function is defined, which is used to assess the computational error in the proposed approach and in the standard numerical approach based on the Runge–Kutta algorithm. Furthermore, an example of a dynamic model of airplane wing flutter on a turbine engine is given to illustrate the applicability of the proposed approach. Numerical solutions show that the proposed approach exhibits high accuracy in mathematical expression, and it is valuable for theoretical research and engineering applications of parametric systems.

1. Introduction

The problem of systems with periodic coefficients occurs in several branches of engineering [1–3], and investigations of response approach and stability are the two most significant dynamic topics in these systems. In the past, the response of systems with periodic coefficients has been studied with several approaches, such as the perturbation method [4], Floquet theory with numerical integration [5], Sinha's numerical computation with shifted Chebyshev polynomials [6,7], David's transfer matrix method [8], linear combination of Floquet eigenvectors [9], and multiple-scales method [10]. These computational formulations can be used to determine the free or forced responses efficiently. To perform the Fourier analysis of the response of systems with periodic coefficients conveniently, a special form of a trigonometric series [11] was introduced to approach the forced response, based on the concept of frequency splitting, which is considerably useful in the area of online vibration monitoring systems and fault diagnosis.

To experimentally validate the theoretical results in a linear parametrically excited system, Han [12,13] and Yeh [14] developed a cantilever beam subjected to electromagnetic stiffness excitation, in which the free response of parametric vibration was observed clearly, and the important dynamic characteristic was confirmed that the spectrum of free response is composed of natural frequency and linear combinations of natural frequency and parametric frequency.

In this study, a free response approach is investigated on a parametric system, and the concept of frequency splitting is used to extend to the problem of free response, which is expressed as a special trigonometric series with an exponential

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damping function. Using algebraic manipulations, the principle oscillation frequency and the free response expression are determined. In addition, an error function is defined to assess the accuracy of the approach, and a real life example is used to illustrate the validity of the proposed approach.

2. Frequency splitting

Consider a single DOF system with viscous damping, excited by a periodic coefficient. Its motion is described as

$$\frac{d^2x}{dt^2} + 2\varsigma\omega_n\frac{dx}{dt} + \omega_n^2(1+\beta\cos\omega_o t)x = 0 \tag{1}$$

which may be rewritten as

$$\frac{d^2x}{dt^2} + 2\varsigma\omega_n\frac{dx}{dt} + \omega_n^2 x = -x\omega_n^2\beta\cos\omega_o t \tag{2}$$

where ω_n is the natural frequency of the system without damping ($\zeta = 0$) and time-varying index ($\beta = 0$).

Based on Eq. (2), a system with periodic coefficients can be schematically drawn as shown in Fig. 1, which can be seen as a special feedback system with frequency splitting. Owing to the frequency splitting in the system, many harmonic components will be driven. When $t \geqslant 0$, the whole physical process of the frequency splitting is described in Fig. 2. The output of this system, $x(t)$, i.e., the free response, can be expressed as a linear combination of the harmonic components related to ω_s and ω_o.

$$x(t) = x_1(t) + x_2(t) \quad = \sum_{k=-\infty}^{\infty} C_k e^{-\varsigma\omega_n t} e^{j(\omega_s + k\omega_o)t} + \sum_{k=-\infty}^{\infty} D_k e^{-\varsigma\omega_n t} e^{-j(\omega_s + k\omega_o)t} \tag{3}$$

where ω_s is a principal oscillation frequency of the free response, and it is not equal to the natural frequency ω_n owing to a feedback loop in the given system.

Because the energy of the free response is distributed in a narrow band of the principal oscillation frequency ω_s, the harmonic coefficients $C_k \to 0$ and $D_k \to 0$ when $k \to \infty$. Therefore, the free response solution for Eq. (2) comes down to the computation of the principal oscillation frequency ω_s and the coefficients of the components C_k and D_k in Eq. (3).

3. Solution

3.1. Characteristic equation

First, substitute $x_1(t)$ from Eq. (3) into Eq. (1), and apply harmonic balance on both sides of the given equation. Then, obtain the following infinite set of linear equations for C_k:

$$\frac{\omega_n^2\beta}{2}C_{k-1} + [\omega_n^2(1-\varsigma^2) - (\omega_s + k\omega_o)^2]C_k + \frac{\omega_n^2\beta}{2}C_{k+1} = 0 \tag{4}$$

$$(k = \cdots - m, -(m-1), \cdots, -3, -2, -1, 0, 1, 2, 3 \cdots m-1, m \cdots)$$

Here, introduce the following notations:

$$\overline{\omega}_k = \omega_n^2(1-\varsigma^2) - (\omega_s + k\omega_o)^2 \tag{5}$$

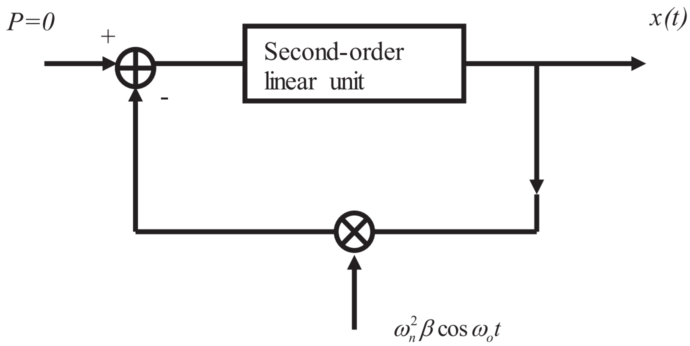

Fig. 1. Special feedback system with frequency splitting, where force excitation $P = 0$.