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Identification of fractional-order systems with time delays using block pulse functions

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ABSTRACT

In this paper, a novel method based on block pulse functions is proposed to identify continuous-time fractional-order systems with time delays. First, the operational matrices of block pulse functions for fractional integral operator and time delay operator are derived. Then, these operational matrices are applied to convert the continuous-time fractional-order systems with time delays to an algebraic equation. Finally, the system's parameters along with the differentiation orders and the time delays are all simultaneously estimated through minimizing a quadric error function. The proposed method reduces the computation complexity of the identification process, and also it does not require the system's differentiation orders to be commensurate. The effectiveness of the proposed method are demonstrated by several numerical examples.

1. Introduction

Fractional calculus (FC) was proposed by Leibniz more than 300 years ago. It is an extension of regular integer order integral and differential calculus to non-integer orders [1,2]. Different from integer calculus (IC), FC is non-local and has the property of long history memory. Research results have been shown that fractional differential equations can describe many real systems more accurately than integer ones. The constitutive behavior of materials [3], thermal diffusion in a wall [4], semi-infinite lossy (RC) transmission line [5], the rotor skin effect of induction machine [6] and viscoelastic systems [7] can be better described using fractional order model.

Recently, much more attention has been paid on the system modeling using fractional order model in the community of control. For example, Podlubny et al. proposed to model the heating furnace using fractional order model (FOM) [8]. Wang et al. built a FOM for a thermal process in a boiler main steam system [9]. In [10], a FOM was built to describe the behavior of a lead acid battery. In [11], a FOM for a solid-core magnetic bearing was built. These encourage research results revealed that FC provides us a power tool to build accurate model for a system, which is very important for high performance controller design.

At present, however, FC has not an acceptable geometrical or physical interpretation, and consequently, there is a great difficulty to build FOM for a system based on mechanism analysis. Therefore, system identification is still the most used approach for building FOM of physical systems. Many researches on fractional system identification have been reported in frequency domain [12–14] and time domain [15–18]. In time domain, the pioneer works on fractional system

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identification came from Lay [15], Lin [16], Cois [17] and Aoun [18], where equation error (EE)-based and output error (OE)-based methods had been proposed. In EE-based method, the model parameters of fractional order system were identified using classical least squares (LS) method under the assumption that the differentiation orders are *prior* known. It involves the computation of fractional derivatives of input-output signals from sampled data. To avoid amplifying noise contained in input-output signals, a linear transformation (low pass filter) was applied into input-output signals, for example, Poisson's state variable filter (SVF) [19]. To eliminate the bias of LS method, the simplified refined instrumental variable for continuous-time systems (SRIVC) method was proposed in [20] to identify FOM. The OE-based method can simultaneously estimate the differentiation orders and model parameters through minimizing the output error between true system and the identified system iteratively [18]. However, to avoid ill conditions in the iterative optimization process, *the fractional differentiation orders are limited to be commensurate*. Furthermore, fractional orthogonal basis functions such as fractional Laguerre basis, Kautz basis were also proposed to identify fractional order systems [21,22].

It is noted that the existing works paid a little attention to the identification of fractional order systems with time delays. Time delay exists in various engineering systems such as chemical process, long transmission lines in pneumatic systems, nuclear reactor, and rolling mill. Fractional order systems with time delays can also be found in practice. For example, the dynamic behavior of HIV infection of CD^+ T-cells [23,24] and motion control systems with actuator limitation [25] can be well modeled using fractional systems with time delays. Therefore, it is significant to develop methods for the identification of fractional order systems with time delays.

In [26], a subspace-based method was proposed to identify fractional order systems with time delays. When the fractional differential orders and time delays are fixed, the parameters were estimated using subspace identification method, whereas the orders and time are estimated using simulated annealing algorithm. In [27], a method for identifying fractional order systems with time delays was proposed in frequency domain. A combination of harmony search and LS method was proposed to identify the parameters, the differentiation order and time delays. Specifically speaking, the optimal differentiation orders and the time delays of the system are found using harmony search, while parameters are obtained by solving a linear LS problem for every differentiation order and time delay generated by harmony search. In [28], the identification of continuous-time commensurate fractional models with time delays was studied. A new linear filter was introduced to the identification process, making the delay term appear as an explicit parameter to be estimated along with other model parameters. Consequently, the model parameters and time delay can be simultaneously estimated using LS method if the fractional differentiation orders are known, otherwise, a nested loop optimization is used.

Above all, no matter with or without time delays, there are several limitations for the identification of fractional order systems. Firstly, the fractional differentiation orders must be known or commensurate, which are usually not available in practice. Secondly, model parameter identification usually involves calculation of fractional derivative of input and output signals, which is an intensive computational burden due to the long memory of fractional calculus.

In this paper, we focus on the identification of fractional order systems with time delays. A novel method is proposed based on block pulse functions operational matrix representation of fractional integral operator and delay operator. The advantage of using operational matrix representation of fractional integral operator and delay operator is that they convert an integral operation to an algebraic one. As a result, the computational complexity is reduced, and most importantly, it can make the differentiation orders and time delays appear as explicit parameters. Therefore, the model parameters, differentiation orders and time delays can be identified simultaneously.

The remainder of the paper is organized as follows. In Section 2, a brief mathematical background of fractional calculus and piecewise orthogonal functions with generalized and delay operational matrices are introduced. In Section 3, the proposed parameter identification method based on operational matrix is presented. In Section 4, numerical examples are given. The conclusion remarks are drawn in Section 5.

2. Mathematical background

2.1. Definitions of fractional derivatives and integrals

The fractional derivatives and integrals operators ${}_aD_t^\alpha$ is defined as

$$
{}_aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \alpha > 0 \\ 1, & \alpha = 0\,, \\ \int_a^t (d\tau)^{-\alpha}, & \alpha < 0 \end{cases} \tag{1}
$$

where a and t are the limits and $\alpha(\alpha \in R)$ is the order of the operation [1]. Unlike integer calculus, there are several different definitions for fractional calculus. The most used definitions are Grünwald-Letnikor (G-L) definition and Riemann-Liouville (R-L) definition. The G-L definition is given as

$$
{}_aD_t^\alpha f(t) = \lim_{h\to 0} \frac{1}{h^\alpha} \sum_{j=0}^{[t-a]/h} \binom{\alpha}{j} f(t - jh), \tag{2}
$$

where $[\cdot]$ denotes the integer parts and h is the finite sampling interval, and