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Stress update algorithm for enhanced homogeneous anisotropic hardening model

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Abstract

A stress integration algorithm is provided for a novel homogeneous-yield-function-based anisotropic hardening (HAH) model. The new model is an extension of the original HAH model that describes cross-hardening or softening of a sheet metal under an orthogonal strain path change. A semi-explicit integration scheme for the stress update is utilized to efficiently handle the gradient of the distorted yield surface during complex strain path changes, as originally proposed by Lee et al. (2012). Validations of the algorithm developed were conducted by comparing the predicted stress–strain curves of dual-phase (DP) 780 and extra-deep-drawing-quality (EDDQ) steels with experimental stress–strain responses observed under cross-loading conditions. Finally, the accuracy of the proposed finite element (FE) formulations was assessed by r-value prediction and preparation of iso-error maps.

Keywords: Anisotropic hardening; Stress update algorithm; Cross-loading; Yield surface

1. Introduction

In the metal forming and shaping processes, materials experience complex loading histories. Numerous experimental and computational approaches to characterizing the mechanical responses of sheet metals under complex non-proportional loading conditions have been used to describe non-proportional strain path changes [1–5]. For example, the Bauschinger effect and transient hardening behavior feature unique stress–strain responses under forward–reverse loading conditions, which is important in the analysis of springback.

The classical isotropic and kinematic hardening laws do not represent these material behaviors under the reversed loading path change. However, the combined isotropic–nonlinear kinematic hardening models have been used successfully to capture the reversed stress–strain curve in an efficient manner [6–8]. Other approaches to describing the microscopic changes have also been proposed. Teodosiu and Hu [9] developed a dislocation-structure-based model combined with kinematic hardening to describe material behavior under strain path changes. Another approach based on texture analysis can be used to predict complex material behavior during multi-path loading [10,11].

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Recently, the authors of the present paper proposed an alternative approach to representing the effect of a strain path change on the mechanical responses of sheet metals, based on the homogeneous-yield-function-based anisotropic hardening [12]. The HAH model does not involve kinematic hardening but rather employs distortion of yield loci as a novel scheme. The model is able to explain most complex hardening responses under the reverse loading. Numerical formulations for use in the FE analyses were provided by Lee et al. [13,14] and many simulations for the prediction of springback were successfully performed [15].

Special phenomena that occur during non-proportional loading, such as hardening stagnation, over-shooting and cross-loading behavior have been observed for various materials in many research studies [2,16–21]. In recent experimental work by Ha et al. [16], unusual overshooting of the flow stress for EDDQ steel was observed when the strain path change was nearly orthogonal and for cross-loading conditions. However, for the same loading conditions, DP steel, an advanced high-strength steel, exhibited the Bauschinger-like transient behavior. Based on these experimental observations, the original HAH model was improved to describe the cross-loading behavior of steels [22,23]. This new model is hereinafter referred to as the "enhanced HAH model".

To implement rate-independent constitutive models in finite element analysis, the sophisticated numerical integration algorithms are necessary to take the efficiency, accuracy, and robustness of the solution into account. The predictor–corrector algorithm (called return mapping) is the most popular among such numerical integration algorithms. Wilkins [24] developed the return mapping method, which consists of two steps. In the first step, a trial stress is calculated based on the elasticity theory. The second step is the corrector step in which the stress is updated in accordance with a flow rule and a consistency condition. Owing to its fully implicit nature and unconditional stability, the closest-point projection method was applied to various elastoplastic constitutive models [25–27]. Ortiz and Simo [28] proposed a general convex cutting plane method that offers simplicity and efficiency in numerical computation. However, because this method produces a conditionally stable solution, special attention is necessary [29]. Yoon et al. [30] developed a multi-stage return mapping method to address the convergence issues that exist when complex constitutive models were used. As an alternative to the return mapping method, sub-stepping stress integration algorithms using the forward Euler method were developed [31–35]. The sub-stepping algorithm was found to be a reliable approach to increasing the accuracy of the stress integration method.

The main purpose of this study was to implement the enhanced HAH model into a finite element analysis using an appropriate stress integration algorithm. Section 2 presents a summary of the enhanced HAH model. Section 3 presents a stress update scheme for the new constitutive model, based on the general convex cutting plane algorithm, which takes the special nature of the enhanced HAH model into account. To avoid arriving at an unstable solution, the sub-stepping algorithm is combined with the general convex cutting plane algorithm. Section 4 presents a validation of the capabilities of the enhanced HAH model that is conducted by determining the plastic response of DP and EDDQ steels under cross-loading conditions using simulation. Section 5 presents the results of an evaluation of the accuracy of the proposed stress algorithm.

2. HAH formulation

2.1. Overview of HAH model

The original HAH model describes distortional yield locus as a function of plastic deformation [12]. The model is expressed as the first degree of a homogeneous function with respect to the deviatoric stress, as follows:

$$\Phi(\boldsymbol{\sigma}) = \left(\phi^q + \phi_h^q\right)^{\frac{1}{q}} = \left(\phi^q + f_1^q \left|\hat{\mathbf{h}}^{\mathbf{s}} : \mathbf{s} - \left|\hat{\mathbf{h}}^{\mathbf{s}} : \mathbf{s}\right|\right|^q + f_2^q \left|\hat{\mathbf{h}}^{\mathbf{s}} : \mathbf{s} + \left|\hat{\mathbf{h}}^{\mathbf{s}} : \mathbf{s}\right|\right|^q\right)^{\frac{1}{q}} = \bar{\sigma}\left(\bar{\varepsilon}\right) \quad (1)$$

where $\mathbf{s}$ is the deviatoric stress of $\boldsymbol{\sigma}$, q is a constant exponent, and $\bar{\sigma}\left(\bar{\varepsilon}\right)$ is the flow curve. The stable component ϕ can be any type of yield function and the fluctuating component ϕ_h represents yield surface distortion. The microstructure deviator $\hat{\mathbf{h}}^{\mathbf{s}}$ is the main variable of the model; it captures the strain path history and determines the direction of the yield surface distortion. The microstructure deviator $\hat{\mathbf{h}}^{\mathbf{s}}$ is the normalized quantity of the tensor $\mathbf{h}^{\mathbf{s}}$, defined as follows:

$$\hat{h}_{ij}^s = \frac{h_{ij}^s}{\sqrt{\frac{8}{3} h_{kl}^s h_{kl}^s}} \quad (2)$$