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A comparison of model reduction techniques based on modal projection for structures with frequency-dependent damping

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ABSTRACT

Frequency response functions are often used in the design of damped structures to assess the level of vibration and evaluate the performances of a damping treatment. Rather than evaluating frequency sweeps on a large-scale finite element system, the frequency response functions can be efficiently computed through model reduction techniques. This paper reviews and compares the reduction techniques based on modal projection which have been developed for structures with frequency-dependent damping, such as structures treated with constrained viscoelastic layers. The insight obtained by this comparison allows to make a motivated choice for a particular model reduction technique. All reviewed methods are quantitatively compared on two illustrative examples.

1. Introduction

Limiting resonant or nearly resonant vibrations is classically achieved through the application of damping treatments such as constrained viscoelastic layers [1–5]. An important tool in the design of such structures is the development of predictive models for numerical simulation, which makes use of the finite element approach. In particular, the finite element discretisation of the differential equations of a problem consisting in a vibrating structure with frequency-dependent damping results in the following equations of motion:

$$(\mathbf{K}^*(\omega) - \omega^2\mathbf{M})\mathbf{U}^*(\omega) = \mathbf{F} \tag{1}$$

where ω is the angular frequency, $\mathbf{K}^*(\omega)$ is the complex, frequency-dependent, symmetric and positive semi-definite stiffness matrix, $\mathbf{M}$ is the real, constant, symmetric, positive definite mass matrix, $\mathbf{F}$ is the real and constant externally applied force amplitude vector and $\mathbf{U}^*(\omega)$ is the complex displacement amplitude vector at the angular frequency ω. This general expression of the equations of motion is consistent with most of the classical damping models (hysteretic, viscous, viscoelastic).

In structural design problems, the geometry, component properties and positioning of the damping treatments are optimised to achieve better performance. The influence of some environmental and/or operating parameters on the efficiency of a damping treatment needs also to be determined. For instance, the viscoelastic materials used in some passive systems have their mechanical properties varying with frequency, temperature, prestrain, ... The efficiency of a damping treatment is generally assessed using frequency response functions. Therefore, to keep the design stage as short as possible, there is a need for reliable computational techniques capable of predicting the frequency response functions of structures with

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frequency-dependent damping at low cost. The direct method consisting in solving Eq. (1) for a large number of frequency steps can be used to evaluate frequency response functions but this approach is computationally expensive. In particular, structures with viscoelastic layers require large order finite element models to get realistic predictions of the dynamic behaviour. It makes the direct approach incompatible with optimisation.

Instead, several methods have been developed to reduce the computational cost of frequency response estimation [6,7]. Many of the reduction methods which are proposed in the literature rely on the assumption that an approximation $\mathbf{U}_r^*$ of the solution $\mathbf{U}^*$ can be constructed in a subspace of reduced dimension spanned by the columns of a reduction basis $\mathbf{T}$:

$$\mathbf{U}^* \approx \mathbf{T}\mathbf{U}_r^* \tag{2}$$

A projection of the matrix system from Eq. (1) on a reduction basis reduces the system size to be solved and an approximated solution is obtained with important computational gain. One class of reduction methods, which will be referred to as modal-based reduction methods, consists in using vibration modes, representing the dynamics of the structure, in the reduction basis $\mathbf{T}$. The most well-known approach is the mode superposition method [8,9], where a limited number of vibration modes is used to retrieve the dynamic displacement. The assumptions underlying this method are that the system is undamped or lightly damped and that the eigenfrequencies are well separated [9]. This approach gives satisfactory results for structures with viscous or hysteretic damping but for structures with strongly frequency-dependent damping, such as sandwich structures with viscoelastic insertions, numerical difficulties arise and have led to the development of computational variants of the mode superposition method [10–18].

Though extensive literature exists on reduction methods, few studies are dedicated to their review and comparison [6,7,13]. In [7,13], comparative studies of several reduction methods are carried out, but they are focused on the precision of the methods and do not address computational efficiency. In [6] both accuracy and computational cost of the reduction methods are under study but the analysis is dedicated to interpolatory model order reduction techniques. Therefore, the objectives of this paper are twofold. The first one is to review existing modal-based reduction methods for the evaluation of the dynamic response of structures with frequency-dependent damping. The second objective is to compare the precision and the computational cost of those methods with respect to the direct method through two illustrative examples. The insight gained by this comparison helps in making a motivated choice for a particular model reduction technique. The outline of this paper is as follows. Section 2 reviews the classical mode superposition approach and highlights its limitations when applied to structures with frequency-dependent damping. Section 3 describes the different variants of the mode superposition method and their implementations. In Section 4, those methods are applied to two sandwich structures with viscoelastic insertions and a comparison in terms of precision and computational time is made. Two systems of different scales are considered for this comparison in order to assess the influence of the model size on the performance of the modal-based reduction methods. Finally, conclusions are drawn in Section 5.

2. The classical mode superposition method and its limitations

The finite element discretisation of the differential equations of a problem consisting in a vibrating elastic structure results in the equations of motion of the system, which in absence of damping are typically of the form:

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{U}(\omega) = \mathbf{F}, \tag{3}$$

where $\mathbf{K}$ and $\mathbf{M}$ are respectively the stiffness and the mass matrices, $\mathbf{U}(\omega)$ represents the solution vector at the angular frequency ω, containing the unknown displacement of the structure, and $\mathbf{F}$ is the load vector. The generalised eigenvalue problem associated with Eq. (3) is:

$$\left(\mathbf{K} - \omega_k^2\mathbf{M}\right)\mathbf{\Phi}_k = \mathbf{0}, \tag{4}$$

where ω_k and $\mathbf{\Phi}_k$ are respectively the eigenfrequency and the eigenvector, also called normal modes, associated to the mode k ($k \in [1 \dots N]$, with N the size of the system). Orthogonality relationships between eigenmodes of distinct eigenfrequencies exist which, in case of $\mathbf{M}$-normalised eigenmodes, leads to the following relations:

$$\begin{aligned} \mathbf{\Phi}_r^T\mathbf{M}\mathbf{\Phi}_s &= \delta_{rs}, \\ \mathbf{\Phi}_r^T\mathbf{K}\mathbf{\Phi}_s &= \omega_s^2\delta_{rs}, \end{aligned} \tag{5}$$

where δ_{rs} denotes the Kronecker delta function. Because the eigenmodes $\mathbf{\Phi}_k$ are independent and orthogonal, they form a basis $\mathbf{T}$ spanning the N-dimensional space in which the solution vector has a unique expansion:

$$\mathbf{U} = \sum_{k=1}^{N}\mathbf{\Phi}_k\chi_k. \tag{6}$$

The modal coordinates χ_k are solutions of the equation:

$$(\omega^2 - \omega_k^2)\chi_k = \mathbf{\Phi}_k^T\mathbf{F}, \tag{7}$$

which is obtained by projection of Eq. (3) on the basis of normal modes.