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Active assignment of eigenvalues and eigen-sensitivities for robust stabilization of friction-induced vibration

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ABSTRACT

As friction couples tangential and lateral degrees-of-freedom of a structure at contact interfaces, the resulting asymmetric dynamic system is prone to dynamic instability. Using state-feedback control, such a frictional asymmetric system can be stabilized through assigning the system desirable eigenvalues; but uncertainties in system parameters can cause assigned eigenvalues to deviate from desired locations and thus stability may be lost.

This study presents a robust stabilization method that assigns both desirable eigenvalues and their sensitivities and thus render assigned eigenvalues stable and insensitive to perturbations in uncertain contact parameters (the friction coefficient, contact damping, and contact stiffness). This method utilizes receptances of the corresponding symmetric part of the asymmetric system. The optimal control input location is first determined by minimizing the Frobenius norm of the normalized eigen-sensitivity matrix. The normalized eigen-sensitivities indicate that the friction coefficient and contact stiffness intrinsically have similar crucial effects on the stability of the system. To demonstrate the application of the proposed control method, the eigen-sensitivities with respect to only the friction coefficient are assigned. A constrained over-determined least-squares problem is solved to assign both required eigenvalues and eigen-sensitivities. Numerical examples validate the effectiveness of the proposed robust control scheme by Monte Carlo simulations.

1. Introduction

Eigenvalues of a system representing dynamic characteristics of the system are invaluable and their assignment is widely studied in numerous engineering fields, such as control theory [1], vibration suppression [2], electric circuits and finite element model updating [3]. For a vibrating system, its eigenstructure, i.e., eigenvalues and eigenvectors, determines its dynamic response. Therefore, eigenstructure assignment has been extensively investigated for vibration suppression using passive structural modifications and active control [2,4].

For general systems with mass, damping and stiffness matrices that are symmetric, the main objective of eigenvalue assignment is to assign appropriate natural frequencies or anti-resonances. Mottershead and Ram [2] first introduced a receptance-based inverse method for passive and active pole assignment, in which only a few measured receptances were needed and no knowledge of the mass, damping and stiffness matrices was required. Mottershead and his colleagues proposed receptance-based state-feedback and output-feedback schemes to actively assign appropriate poles/zeros to

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symmetric systems [4–6]. For symmetric systems, Datta et al. [7] were the first to study the quadratic partial eigenvalue assignment problem (PEVAP). A multi-input state-feedback control algorithm was proposed for quadratic PEVAP in [8] and the time-delay problem in PEVAP was investigated in [9]. Tehrani et al. [10] applied a receptance method-based PEVAP to a real lightweight glass-fibre beam. Based on the hybrid method for single-input vibratory systems put forward by Ram et al. [11], Bai et al. [12] developed a multi-step hybrid method for PEVAP of multi-input vibratory systems. In addition, eigenvalue sensitivities were successfully assigned based on receptance measurements in [13].

However, if a system includes certain nonconservative internal forces, the symmetry of its damping and stiffness matrices may be lost. Such a nonconservative system with asymmetric system matrices is likely to have eigenvalues with positive real parts, and thus is prone to instability. Friction, as a kind of nonconservative force, can induce unstable self-excited vibration. In real systems with friction, large amplitude vibration appears and usually noise is generated simultaneously [14], such as, wheel-rail noise [15], machine tool chatter [16] and vehicle brake squeal [17]. To stabilize an unstable asymmetric system, Ouyang et al. assigned eigenvalues of the system to the left half of the complex plane using both structural modifications [18] and active state-feedback control [19]. Singh and Ouyang [20] considered a constant time-delay in the feedback control loop for assigning stable eigenvalues to second-order damped asymmetric systems. Tehrani and Ouyang [21] proposed a partial pole assignment method for asymmetric systems.

The robustness of eigenvalue assignment for both symmetric and asymmetric systems has attracted much attention. For symmetric systems, the well-known study of Kautsky et al. [22] measured robustness using the condition number of the eigenvector matrix of the first-order system. The linear matrix inequality (LMI) approach was used to study robust pole placement for the first-order [23] and second-order [24] linear systems subject to system uncertainties or structural failures. Tehrani et al. [25] presented a sequential multi-input state-feedback method that minimized the norm of the eigen-sensitivities with respect to measurement errors in receptances. Recently, a modified Schur method was proposed for robust eigenvalue assignment in state-feedback control [26]. Araújo et al. [27] assessed eigen-sensitivity to parameter variation in second-order linear systems under state feedback and state derivative feedback. For asymmetric systems with friction, the robustness is of vital importance. Since a large class of friction-induced vibration (FIV) problems is sensitive to variability in friction, material properties, geometry of sliding surfaces, normal load, etc. [28], it is difficult to be suppressed [29]. Uncertainties in FIV can arise from different sources, for example, from friction laws themselves [30] or from surface roughness of friction materials [31]. It is preferable to identify the most influential parameters on FIV. For example, Nechak et al. [32] ordered parameters in terms of their impacts and only considered influential parameters to form a Kriging model for robust and optimal design of brakes against dynamic instability.

This research focuses on developing a robust stabilization method for a class of FIV problem when the contact parameters are uncertain. The FIV is manifested as a typical asymmetric, linear time-invariant dynamic system. The uncertain contact parameters are assumed as independent normally distributed random variables. The proposed robust method takes advantage of the receptance method that requires no detailed information of the mass, damping and stiffness matrices. Eigen-sensitivities with respect to uncertain contact parameters are derived using a perturbation method. By minimizing the Frobenius norm of the normalized sensitivity matrix formed by eigen-sensitivities, the optimal control input position is determined which results in the least scatter in assigned eigenvalues. Through examining the real parts of the normalized eigen-sensitivities, the friction coefficient and the contact stiffness are found to intrinsically have similar crucial effects on the stability of the system. To ensure the assigned eigenvalues to be stable and insensitive to the uncertain contact parameters, a constrained over-determined least-squares problem is proposed. By solving this optimization problem, both required eigenvalues and their normalized eigen-sensitivities with respect to critical uncertain parameters are assigned. Finally, Monte Carlo simulations are conducted to verify that the proposed approach is effective and robust.

2. Receptance-based robust state-feedback control

2.1. Basic theory

The dynamic equation of a general asymmetric vibrating system with single-input state feedback u in Laplace space is formulated as

$$[\mathbf{M}s^2 + (\mathbf{C}_s + \mathbf{C}_{as})s + (\mathbf{K}_s + \mathbf{K}_{as})]\mathbf{x}(s) = \mathbf{p}(s) + \mathbf{b}u(s) \tag{1}$$

and the single-input u formed with displacement and velocity feedbacks is given as

$$u(s) = -(\mathbf{g} + s\mathbf{f})^{\mathrm{T}}\mathbf{x}(s) \tag{2}$$

In Eq. (1), $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{n\times n}$ are the mass, damping and stiffness matrices, respectively. $\mathbf{x}(s)$ and $\mathbf{p}(s) \in \mathbb{R}^n$ are respectively the Laplace transforms of the displacement vector and the external force vector. $\mathbf{b} \in \mathbb{R}^n$, indicating the positions where the control input $u(s)$ is received, consists of element 1 or 0; For general structural systems, $\mathbf{M} = \mathbf{M}^{\mathrm{T}}$ is positive-define, $\mathbf{C}_s = \mathbf{C}_s^{\mathrm{T}}$ and $\mathbf{K}_s = \mathbf{K}_s^{\mathrm{T}}$ are semi positive-define; Specifically, $\mathbf{C}_{as} = \sum_{k=1}^{m}\mu_k c_{ck}\mathbf{E}_k$ and $\mathbf{K}_{as} = \sum_{k=1}^{m}\mu_k k_{ck}\mathbf{E}_k$ are the asymmetric matrices induced by contact damping and contact stiffness at frictional contact interfaces. μ_k, c_{ck}, k_{ck} are the contact parameters,