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A hybrid nonlinear vibration energy harvester

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ABSTRACT

Vibration energy harvesting converts mechanical energy from ambient sources to electricity to power remote sensors. Compared to linear resonators that have poor performance away from their natural frequency, nonlinear vibration energy harvesters perform better because they use vibration energy over a broader spectrum. We present a hybrid nonlinear energy harvester that combines bi-stability with internal resonance to increase the frequency bandwidth. A two-fold increase in the frequency bandwidth can be obtained compared to a bi-stable system with fixed magnets. The harvester consists of a piezoelectric cantilever beam carrying a movable magnet facing a fixed magnet. A spring allows the magnet to move along the beam and it provides an extra stored energy to further increase the amplitude of vibration acting as a mechanical amplifier. An electromechanically coupled mathematical model of the system is presented to obtain the dynamic response of the cantilever beam, the movable magnet and the output voltage. The perturbation method of multiple scales is applied to solve these equations and obtain approximate analytical solutions. The effects of various system parameters on the frequency responses are investigated. The numerical approaches of the long time integration (Runge-Kutta method) and the shooting technique are used to verify the analytical results. The results of this study can be used to improve efficiency in converting wasted mechanical vibration to useful electrical energy by broadening the frequency bandwidth.

1. Introduction

The growth of ultra-low-power sensor technologies inspires the use of alternative energies. One of the most ubiquitous sources of energy is ambient mechanical vibration, which can be converted to useful electrical energy. This renewable source of energy can replace batteries, which have short lives and high maintenance costs. Most of the current vibration energy harvesters are linear resonators that have a narrow bandwidth, which causes a significant drop in the output once the excitation frequency differs from the resonant frequency. As ambient vibrations have a wide spectrum at the low-frequency range, that defect defeats attempts to use linear resonators as convenient energy harvesters [1].

Increasing the frequency bandwidth of resonators will increase the efficiency of converting mechanical energy to electricity. To increase the bandwidth, several methods introduced nonlinearities into the energy harvesting system [2] such as using stoppers to realize a spring hardening effect [3–5] and using axial static preload to stiffen or soften the structure [6,7]. Researchers also applied parametric excitation to trigger nonlinearity for energy harvesting. Yildirim et al. [8] recently

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Nomenclature

D_3	electric displacement
e_{31}	piezoelectric constant
S_1	mechanical strain
ϵ^s_{33}	piezoelectric material permittivity constant
E_z	electric field
T_{1s}	mechanical stress
c_{11}	compliance of piezoelectric
$\lambda(t)$	flux linkage
L_p	length of piezoelectric
b_p	width of piezoelectric
h_p	thickness of piezoelectric
L	length of beam
b	width of beam
h_s	thickness of beam
L	length of entire piezoelectric beam
A	cross sectional area of entire piezoelectric beam
ρ	volume density of entire piezoelectric beam
I	moment of inertia
E	modulus of elasticity
k	stiffness of spring
$w(x,t)$	lateral deflection of the beam
$s(t)$	position of movable magnet measured from the fixed support
$S(t)$	position of movable magnet measured from equilibrium position
s_e	equilibrium position of movable magnet
k_1	normalize first natural frequency of cantilever beam
U_{mag}	magnetic potential energy
y	base excitation
β	slope angle of the beam deflection curve with horizontal line
R	resistance load
$\phi_1(x)$	the first mode shape for cantilever beam
$\alpha(t)$	dynamic response of cantilever beam
F_{magx}	magnetic force tangential to the beam length
F_{magy}	magnetic force normal to the beam length
μ_0	permeability of space
d	distance between the tip of the cantilever and fixed magnet
D	horizontal distance between two magnets
N	magnetization moment of magnet
ω_s	natural frequency of the spring
m	mass of movable magnet
s_0	original length of spring
μ_n	damping terms
$v(t)$	voltage
T_0, T_1	two time scales
ϵ	scaling parameter
P_1, P_2	complex variables
$E(T_1)$	a variable
σ_1, σ_2	small detuning parameters
Ω	frequency of excitation

designed a clamped-clamped beam with a movable central magnet inside a coil. The experiment showed frequency softening and a broader bandwidth close to the primary and principal parametric resonances.

Nonlinearity from bi-stable systems can also broaden the frequency bandwidth. Such systems use two magnets (one stationary and one moving) for piezoelectric energy harvesters [9–11] to create a double-well potential function. Their performance was studied under base vibrations of harmonic [11] and random natures [12–16]. Across various excitation levels, bi-stable systems outperformed linear ones. Bi-stable systems also were employed in electromagnetic generators [17,18]. The magnetic force adds cubic stiffness nonlinearity and makes the oscillator a Duffing type with frequency hardening behavior [17,19–22]. Daqaq [23] investigated the response of such harvesters as a unimodal Duffing-type oscillator exposed to White Gaussian and Colored excitation. Combining piezoelectric and electromagnetic energy harvesting mechanisms, a