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On the optimality of harmonic excitation as input signals for the characterization of parameters in coupled piezoelectric and poroelastic problems

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ABSTRACT

In this work we provide theoretical and numerical results regarding optimal designs of experiments with an emphasis on coupled problems like piezoelectrics and poroelastics. The work is motivated by the need of identifying parameters for complex problems from measured data, where it is a priori not clear which data are to choose. We assume a harmonic excitation of the systems and measurements related to different excitation frequencies. Results of optimal experimental designs under harmonic excitations are reviewed and adapted correspondingly, e.g., the D-optimality criterion is extended to the case of multiple fields. The manuscript at hand further reviews techniques to identify parameters and their statistical properties and discusses the previously derived theory for two examples, one coming from piezoelectricity, the other from poroelasticity. For these examples, it is analytically shown that they fit to the previously presented theory. Numerical results discussing the optimal choice of the fields to measure and finding the optimal excitation frequencies finalize this work.

1. Introduction

In many systems, there are couplings of two or even more physical quantities which are influencing each other. For instance, in mechatronical systems, different types of sensors and actuators are developed to convert signals from one physical field into signals of another physical field. Examples are piezo-electrically driven actuators for ultrasound generation, electro-magnetic transducers, micro-electro-mechanical devices and so on. Also in other engineering disciplines we find such coupled problems, e.g., thermal-hydro-mechanically coupled systems which dominate the behavior of structures likes dams or dikes. Generally, the design and analysis of these structures are supported by means of models. A variety of models has been derived, which depending on the application, are formulated in one, two or even three dimensions. Mostly, the effects are described by coupled systems of instationary partial differential equations. Depending on the dimension, but also on the material properties (isotropic or anisotropic), a series of material specific parameters needs to be known in order to make reliable predictions. This leads to inverse problems which are often ill-posed. The determination of the material parameters is the content of many works, where generally discrepancies between measured and simulated signals are systematically reduced by applying techniques of nonlinear optimization or regularizing methods. To organize optimally the

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Design of Experiments (understood as the design of a physical experiment and not the design for a statistical method, as e.g., [1,2]), a series of approaches has been developed in literature where mainly all approaches which assume unbiased estimators are based on the Fisher information matrix, see, e.g. [3] for a detailed work on the parameter estimation of distributed parameter systems. A vital discussion about robustness issues and applicability of model input design for parameter estimation and control of systems is discussed in [4–6]. In many experimental setups, time-harmonic loads are applied to excite the structures and to measure responses in dependency on the current excitation frequency. A set of responses might then be used as input for parameter identification activities in frequency domain, e.g., in piezoelectricity [7] or for dispersive dielectrics [8]. The questions arising within the inverse (one field) problems formulated in frequency domain are:

- At which frequencies should I excite my system, see, e.g. [9]?
- How many different excitation frequencies should I consider at least?
  In a multifield context, an additional question appears.
- How many different field quantities do I need to consider at least?

In the given manuscript we provide answers to all three questions.

For the question about the number of excitation frequencies results have been established in [10–12], where however the applicability of the methods is proven solely for academic types of problems and ordinary differential equations.

The work in [13] extended previous works on ordinary differential equations to one field partial differential equations, however not for coupled problems. We go in this work a step ahead and demonstrate applicability to more complex systems which are mainly characterized by a coupling of two physical quantities. Even though we used simplified representations of the problems considered, the theory and results can be directly transferred to any PDE which can be solved by numerical means. Thus the given discussion is no limitation to cases where only analytical formulations are available. Merely, we show that the theory is indeed applicable by decomposing the solutions. If numerical means are the only possibility to compute the model input-output relations, sensitivity methods as discussed, e.g., in [14] for dynamical situations are to applied to obtain the entries in the Fisher matrices. Additionally, the representation of the Fisher information matrix is justified here firstly with the required rigor by spectral densities of the excitations and derivatives of the Fourier transform of the fundamental solutions of the models considered.

The paper is organized as follows: After the introduction we formulate the forward operator related to the given inverse problems in frequency domain and allow its decomposition into a part corresponding to the transmittance of the problem and another part related to its excitation. By this, we define the Fisher Matrix as the sum of functions of spectral densities and by linking it with the Theorem of Caratheodory (Section 2.1) we establish an upper bound for the number of different excitation frequencies for optimal designs. In subSections 2.3 and 2.4 we reveal optimality criteria and methods which allow the identification of the parameters for the examples presented in Section 3 and their statistical parameters. Section 3 introduces two important examples of coupled problems. These are piezoelectricity and poroelasticity, which are of high importance in their respective fields of research, see, e.g., [15–17]. In the analysis we make use of both one-dimensional fundamental representations of the coupled problems solutions and numerical approximations in higher dimensions. Numerical examples are given for both, the inverse problems and the related optimal design problems. The paper closes with an appendix which justifies the representation of the Fisher information matrix in terms of the spectral density and derivatives of the Fourier transform of the fundamental solution.

2. Forward operators and information matrix

We are making the following assumptions: The experimental time for the systems regarded is large and the excitation signal in time domain allows a spectral representation. Let us assume that the response of a system of coupled field equations is given by

$$F : X \to Y \tag{1}$$

$$p \mapsto F(p, \bar{\omega}) \tag{2}$$

with $X \subseteq \mathbb{R}^r$ being the space of model input parameters, $Y \subseteq \mathbb{C}^{N_n}$ the set of measurements, $p \in \mathbb{R}^r$ a vector of real valued material parameters of length r. F is the response of the system, depending on the parameters p and excitation frequencies $\bar{\omega} = (\omega_1, \dots \omega_n)^T$. We assume that the recorded values are subjected to normally distributed random noise ϵ_i with zero mean and finite variance, i.e.

$$y_i = F_i(p, \bar{\omega}) + \epsilon_i, \quad i = 1, \dots, N_n, \tag{3}$$

where N_n are the number of measurements. If $F(p, \bar{\omega})$ is complex valued, than the noise is added independently on both, the real and imaginary part, with equal variance and zero mean following a normal distribution. According to [18] this yields complex Gaussian distributed random variables with zero mean and the given variance. Consequently, we assume that throughout the work there are no correlations between the data, neither for the real and imaginary part nor for observations from different fields.