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Identification of nonlinear vibrating structures by polynomial expansion in the z-domain

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A new method in the frequency domain for the identification of nonlinear vibrating structures is described, by adopting the perspective of nonlinearities as internal feedback forces. The technique is based on a polynomial expansion representation of the frequency response function of the underlying linear system, relying on a z-domain formulation. A least squares approach is adopted to take into account the information of all the frequency response functions but, when large data sets are used, the solution of the resulting system of algebraic linear equations can be a difficult task. A procedure to drastically reduce the matrix dimensions and consequently the computational cost – which largely depends on the number of spectral lines – is adopted, leading to a compact and well conditioned problem. The robustness and numerical performances of the method are demonstrated by its implementation on simulated data from single and two degree of freedom systems with typical nonlinear characteristics.

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1. Introduction

The research activity in the wide field of nonlinear system dynamics is attested by the continuous publication of numerous books, Ph.D. theses, papers and benchmarks – see for example [1–9]. In particular the nonlinear system identification, just like the sector of modal parameters extraction, exhibits a sort of time–frequency ambivalence. As an example, a time-domain subspace identification algorithm, extended to nonlinear system and named nonlinear subspace identification (NSI), was proposed in [10] and later developed in the frequency domain in [11]. Each implementation has advantages and disadvantages as pointed out in [12], so that it should be advisable to rely on different data processing techniques to investigate structures properties. The objective of the present paper is to present a new frequency domain method for the identification of nonlinear systems, inspired by the basic principle of the nonlinear identification through feedback of the outputs (NIFO) technique introduced in [13]. Each frequency response function (FRF) of the underlying linear system is expressed by a polynomial ratio in the z-domain, shifting the problem from the estimation of the FRFs to the definition of the constant coefficients describing both the linear and nonlinear parts of the system. Poles and zeros of the various FRFs are in other words simultaneously computed with the (assumed frequency independent) parameters characterizing the nonlinear terms, thus achieving a complete identification of the structure in the same step of the procedure. The nonlinear identification by polynomial expansion in the z-domain (NIPEZ) method here proposed can deal with multiple input

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multiple output (MIMO) systems containing different and (possibly) many nonlinearities and is designed in order to efficiently compute a least squares solution, which takes into account the information contained in each FRF.

The paper is organised as follows. Section 2 introduces the theoretical background of the NIPEZ method, describing the procedure in details. Section 3 is devoted to the numerical examples, with single and two degree of freedom systems. Synthetic data sets are corrupted by additive noise to show its influence on estimates of linear and nonlinear parameters. Stabilisation diagrams are used for validating the identification procedure. The conclusions of the study are presented in Section 4.

2. Outline of the method

For a time invariant, viscously damped system with N degrees of freedom and S nonlinearities, the second order time domain equations of motion can be written in the form [13]

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t)+\sum_{s=1}^{S}\mathbf{L}_s\mu_s g_s(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are symmetric square matrices, $\mathbf{x}(t)$ is the displacement column vector, $\mathbf{f}(t)$ is the external forces column vector, $g_s(t)$ indicates the kind of nonlinearity and has to be specified *a priori*, μ_s is the constant parameter of the nonlinear term and vector $\mathbf{L}_s$ specifies its position, with $\mathbf{L}_s=\begin{bmatrix}L_{1s} & \cdots & L_{ks} & \cdots & L_{Ns}\end{bmatrix}^T$ and $L_{k\,s}=0$ or $L_{k\,s}=\pm 1$. In this model any nonlinear contribution is acting as a force whose position and form is entirely defined, while its intensity depends on the unknown constants μ_s.

It is worth noticing that the methodology relies on the feedback of the outputs stated by Eq. (1): the same principle has been used in [10,11,13] which can therefore be assumed as valuable terms of comparison.

The frequency domain model corresponding to Eq. (1) is

$$\mathbf{B}_L(\omega)\mathbf{X}(\omega)=\mathbf{F}(\omega)+\sum_{s=1}^{S}\mathbf{L}_s\mu_s G_s(\omega) \tag{2}$$

where $\mathbf{B}_L(\omega)$ is the linear impedance matrix and $\mathbf{X}(\omega)$, $\mathbf{F}(\omega)$, $G_s(\omega)$ are the Fourier transforms of $\mathbf{x}(t)$, $\mathbf{f}(t)$ and $g_s(t)$ respectively – $\mathbf{L}_s$ and μ_s are constant quantities. In particular it is assumed that the nonlinear terms μ_s do not vary with frequency, which distinguishes the NIPEZ method from both NIFO and NSI techniques [13,10].

With the position $\mathbf{H}(\omega)=\mathbf{B}_L^{-1}(\omega)$, where $\mathbf{H}(\omega)$ is the (linear) frequency response matrix, Eq. (2) becomes

$$\mathbf{X}(\omega)=\mathbf{H}(\omega)\mathbf{F}(\omega)+\sum_{s=1}^{S}\mathbf{H}(\omega)\mathbf{L}_s\mu_s G_s(\omega) \tag{3}$$

which represents the structure of the identification model.

The input and output time histories (appropriately sampled and sufficiently long), the number, kind and position of the nonlinearities are given so that $\mathbf{X}(\omega)$, $\mathbf{F}(\omega)$, $\mathbf{L}_s$ and $G_s(\omega)$ are completely defined.

The question is how the modal parameters (frequency, damping ratio and mode shape, all of them buried in the FRF matrix $\mathbf{H}(\omega)$) of the underlying linear system as well as the μ_s parameters of the nonlinear terms can be extracted.

To explain the proposed NIPEZ procedure we firstly focus the attention on the single input F_p single output X_q (SISO) equation, extracted from Eq. (3) – ω is removed for the sake of simplicity:

$$X_q=H_{qp}F_p+\left(\sum_{s=1}^{S}\mathbf{H}\ \mathbf{L}_s\mu_s G_s\right)_q \tag{4}$$

or also, in an expanded notation

$$X_q=H_{qp}F_p+\mu_1 G_1\sum_{k=1}^{N}H_{qk}\ L_{k1}+\ldots+\mu_S G_S\sum_{k=1}^{N}H_{qk}\ L_{kS} \tag{5}$$

Lacking the nonlinear terms, Eq. (5) would give $X_q=H_{qp}F_p$ as expected. This also reminds us that a sound computation of a FRF, i.e. H_{qp}, is better achieved by adopting one of the usual estimators based on power spectral density (PSD) functions [14] than by simply performing the ratio X_q/F_p. It is then advisable to multiply Eq. (5) by the complex conjugate output X_q^* so to pave the way for the computation of PSD functions by means of the Welch periodogram:

$$X_q^*X_q=H_{qp}X_q^*F_p+\mu_1 X_q^*G_1\sum_{k=1}^{N}H_{qk}\ L_{k1}+\ldots+\mu_S X_q^*G_S\sum_{k=1}^{N}H_{qk}\ L_{kS} \tag{6}$$

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