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Nonlinear vibrating system identification via Hilbert decomposition

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ABSTRACT

This paper deals with the identification of nonlinear vibration systems, based on measured signals for free and forced vibration regimes. Two categories of time domain signal are analyzed, one of a fast inter-modulation signal and a second as composed of several mono-components. To some extent, this attempts to imitate analytic studies of such systems, with its two major analysis groups – the perturbation and the harmonic balance methods.

Two appropriate signal processing methods are then investigated, one based on demodulation and the other on signal decomposition. The Hilbert Transform (HT) has been shown to enable effective and simple methods of analysis. We show that precise identification of the nonlinear parameters can be obtained, contrary to other average HT based methods where only approximation parameters are obtained. The effectiveness of the proposed methods is demonstrated for the precise nonlinear system identification, using both the signal demodulation and the signal decomposition methods.

Following the exposition of the tools used, both the signal demodulation as well as decomposition are applied to classical examples of nonlinear systems. Cases of nonlinear stiffness and damping forces are analyzed. These include, among other, an asymmetric Helmholtz oscillator, a backlash with nonlinear turbulent square friction, and a Duffing oscillator with dry friction.

1. Introduction

1.1. Hilbert transform (HT) identification in time domain

This paper, addressing the identification of nonlinear vibrating systems, is, in some sense, an extension of research on prior identification methods [1]. Named FREEVIB and FORCEVIB they enabled the reconstruction and separation of the *average* nonlinear elastic and the friction force in the equation of motion of an SDOF system, even under noisy vibration. The FORCEVIB method enabled to estimate the modal parameters for a fast sweeping excitation, including just only several full cycles of the sweeping excitation. Contrary to traditional forced vibration testing of vibration systems, requiring more time and a slower sweeping frequency, the HT identification methods were less time consuming than traditional spectral analysis techniques. Using these HT methods in the time domain, it was shown how to extract both the instantaneous undamped frequency and also the average nonlinear elastic force characteristics.

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However both these mentioned HT methods were used to just detect the low-pass filtered instantaneous amplitude and frequency of signals. The detected instantaneous functions were presented as smooth and slow varying functions of time. They had no fast modulations, nor did they consist of a number of multiple harmonics. Thus the instantaneous amplitude and frequency did not require demodulation or decomposition procedures.

This paper's main contribution is showing the logic and developing an approach, whereby the solutions of a nonlinear vibration system is represented either by an expansion of components with different frequencies, or, alternatively by a time varying signal with an oscillating instantaneous frequency and envelope. Thus two more modern and precise time domain methods are first presented together, both applying the HT.

The first of these methods, the Demodulation based one, follows the time varying amplitudes and frequencies. The second method is based on signal Decomposition, with fundamental and harmonics of integer multiple frequencies. Contrary to the prior mentioned detection methods, *precise* and not just *averaged* system skeleton curves can then be identified by each of the demodulation or decomposition methods.

1.2. Categorization of approaches

Nonlinear vibration analysis of mechanical systems involves analytical modeling, leading to some nonlinear ordinary differential equations, and often necessitating various computational aspects. Engineering approaches often involve analysis via actual measurements, and hence the use of appropriate signal processing techniques. It is certainly appropriate to attempt to relate and categorize established analytical methods to the signal processing ones used in practical measurements analysis. We surely will not claim to have developed an actual unifying approach. However some similarities between known analytical approaches and the signal processing ones used in this paper are interesting, to say the least.

Of the many analytical approaches developed for quantitative analysis of the dynamic behavior of nonlinear systems, the categorization by [2] into two main groups seems especially relevant. The first group includes the methods of multiple time scaling and the Krylov–Bogoliubov–Mitropolsky (KBM) approach [3]. This group of methods can be used to derive a set of simple differential equations, usually described in terms of the *varying amplitudes* and phases of motions. The second group includes *harmonic balancing* and the Galerkin and Ritz procedures [4]. This group of methods can be employed to determine directly asymptotic periodic and quasi-periodic solutions.

The KBM perturbation method (or method of averaging) plays a powerful role in describing the long time behavior of the solution. The essential idea of the method consists of varying the amplitude so slowly that no secular terms can arise in the solution. Thus due to the averaging principle, the exact differential equation of the motion is replaced by its averaged version. When a weakly nonlinear autonomous oscillator is described by an ordinary differential equation

$$\ddot{u} + \omega^2 u = \varepsilon f(u, \dot{u}). \tag{1}$$

its simple harmonic solution can be written in the form $u = A\sin(\omega t + B)$, where the natural frequency ω is known but the amplitude$A$and the modulation frequency addition B are not known. The solution to the perturbed equation is assumed to take the same form, but now A and B are allowed to vary with time and it is assumed also that $\dot{u} = A(t)\omega\cos(\omega t + B(t))$. Thus, for unknown functions A and B, slowly varying with respect to time when compared to ε, their dependence on the phase in each period can be (approximately) removed by averaging the right hand side of (1)$u_0(t, \varepsilon) = A_0(t, \varepsilon)\sin(\omega t + B_0(t, \varepsilon))$. As a result the amplitude and frequency of the approximate solution as well as their

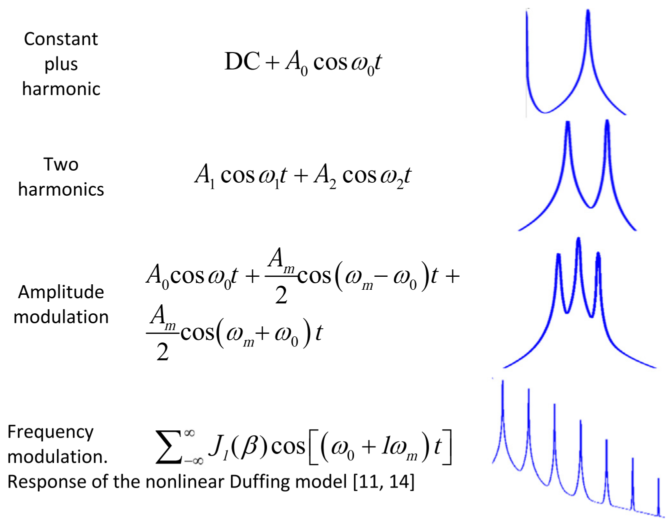

Fig. 1. Typical vibration signals.