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Multiscale structural topology optimization with an approximate constitutive model for local material microstructure

Liang Xia*, Piotr Breitkopf

Laboratoire Roberval, UMR 7337 UTC-CNRS, Université de Technologie de Compiègne, Centre de Recherches de Royallieu, CS 60319, 60203, Compiègne Cedex, France

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Highlights

- An approximate constitutive model is built for local material microstructure using a database model.
- The database is further reduced into a separated representation by means of tensor decomposition.
- The approximate constitutive law can well serve structural design with a reduced computational cost.

Abstract

This paper builds on our recent work (Xia and Breitkopf, 2014) on multiscale structural topology optimization where at the microscopic scale, local materials are optimized concurrently according to current loading status. The former design framework requires intensive computational cost due to large number of repetitive local material optimizations. To circumvent this limitation, in the present work, we construct a reduced database model viewing the local material optimization process as a generalized constitutive behavior using separated representations. In this model, the database is built from a set of numerical experiments of local material optimizations in the macroscopic strain tensor space. Each value in the database corresponds to the strain energy density evaluated on a material microstructure, optimized according to the imposed macroscopic strain. By tensor decomposition, a continuous representation of the strain energy density is built as a sum of products of one dimensional interpolation functions. As a result of this *a priori* off-line step, the effective strain–energy and stress–strain relations required for macroscopic structural evaluation and optimization are provided in a numerically explicit manner. The results given by the reduced database model are compared with full-scale results. It is also shown that this explicit constitutive behavior representation can well serve multiscale structural design at a significantly reduced computational cost.

Keywords: Separated representation; Model reduction; Homogenization; Multiscale analysis; Topology optimization

1. Introduction

The increasing use of heterogeneous materials requires investigation at the local material scale when analyzing structural performances. Meanwhile, the fast progress in the field of material science, allows manufacturing materials

* Corresponding author.
E-mail address: liang.xia@utc.fr (L. Xia).

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with space-varying properties [1]. In pursuing higher-performance structures, conventional optimization with homogeneous materials assumed at the local scale is no longer satisfying and local scale materials need to be optimally designed in a concurrent or integrated manner. With such model, one encounters two coupled design problems working at different scales.

Taking topology optimization as an example, most of the existing research focuses on an individual scale, either designing structures made of homogeneous materials [2], or designing materials for expected effective performance [3]. A usual strategy applied to bridge the two scales is designing an universal material microstructure at the local scale either for a fixed [4,5] or concurrently changed [6] structure at the macroscopic scale. Obviously, such designs have not yet released full potentiality of multiscale analysis. A further step is made in [7] by designing several different cellular materials for a layered structure following a two-step design procedure. In fact, a pioneer attempt to the topic traces back to [8], where simultaneous optimal designs are performed for both structure and cellular materials following a decomposed design procedure given in even earlier works [9,10]. This work has later been extended to 3D [11]. Note that, due to the intensive computational cost, though it was assumed that cellular materials vary from point to point at the macroscopic scale in [8,11], in practice they are defined in an element-wise manner. Another more specific type of concurrent design can be found in [12,13] for structural topology and composite laminate orientation designs of composite structures.

Concerning multiscale structural design, cellular materials at the local scale are optimized in response to the displacement solution at the macroscopic scale while the optimized cellular materials in turn modify the macroscopic constitutive behavior. The equilibrium problem at the macroscopic scale is therefore in general nonlinear. Such nonlinearity has been neglected in early works [8,11] for reasons that both scale design variables were updated simultaneously and no converged local material design solutions were required for macroscopic structural equilibrium.

In order to address this nonlinearity due to the adaptation of local materials, a concurrent design framework using an FE^2-based iterative resolution is developed in our recent work [14]. It is shown that though linear models are assumed at both scales, the macroscopic structural equilibrium is nonlinear due to the optimization or adaptation of local materials. The FE^2 scheme is a widely used multiscale analysis model which is able to evaluate the macroscopic structural responses with an accurate account of local material nonlinearities [15,16]. A recent related work [17] has also employed FE^2 scheme to address the concurrent topology design, where hyperelasticity is considered at the local material scale.

The FE^2-based concurrent design framework [14] requires intensive computational cost due to large number of instant local material optimizations. To limit the computing time, a straightforward strategy is using parallel computing [17,18] because local material optimization tasks are independent. Alternatively, reduction in computational cost can be achieved by using Reduced Order Models (ROMs) [19,20]. As can be observed in [14], multiple loops of similar or even repeated computations are performed at the local scale during the design process. We have shown the benefit of using ROM within an optimization design framework in [21], where a bi-level reduced ROM is developed for the design of multiscale structure with a constant nonlinear microstructure defined at the local material scale. Note that, by simultaneous use of parallel computing and ROM [22], a further reduction in computational time can be achieved in multiscale analysis [23]. Most of the existing ROMs constructed using reduction strategies such as Proper Orthogonal Decomposition (POD) [24,25], Proper Generalized Decomposition (PGD) [26–28], material map model [29,30], eigendeformation-based reduction [31,32], Nonuniform Transformation Field Analysis (NTFA) [33–36], and Numerical EXplicit Potentials (NEXP) [37,38] are developed intending to represent the effective constitutive law of nonlinear heterogeneous materials.

In the present work, in viewing the local material optimization process as a generalized constitutive behavior, we make a step further and adapt the NEXP model to represent this new regime of material nonlinearity. By this model, we construct firstly a database by means of a set of numerical experiments to describe the effective strain energy density function in a test space of macroscopic strain tensor. Each value of the database corresponds to the strain energy density evaluated for a material microstructure, topologically optimized according to the imposed macroscopic strain. By tensor decomposition, a continuous representation of the strain energy density function is built as a sum of products of one-dimensional interpolation functions. As a result of this *a priori* off-line step, the effective strain–energy and stress–strain relations required for macroscopic structural evaluation and optimization are provided in a numerically explicit manner. The explicit material behavior representation given by the reduced database model is then used to serve the concurrent design [14] at a negligible computational cost.

The remainder of this paper is organized in the following manner: Section 2 reviews the concurrent design framework [14]; Section 3 gives the generalized constitutive behavior of locally optimized materials; Section 4 presents