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Identification of systems containing nonlinear stiffnesses using backbone curves

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ABSTRACT

This paper presents a method for the dynamic identification of structures containing discrete nonlinear stiffnesses. The approach requires the structure to be excited at a single resonant frequency, enabling measurements to be made in regimes of large displacements where nonlinearities are more likely to be significant. Measured resonant decay data is used to estimate the system backbone curves. Linear natural frequencies and nonlinear parameters are identified using these backbone curves assuming a form for the nonlinear behaviour. Numerical and experimental examples, inspired by an aerospace industry test case study, are considered to illustrate how the method can be applied. Results from these models demonstrate that the method can successfully deliver nonlinear models able to predict the response of the test structure nonlinear dynamics.

1. Introduction

With the continual drive for more efficient and lightweight structures, increasingly nonlinear structural dynamics are being observed in industrial applications. This presents a problem when developing accurate dynamic models of the structure as conventional parameter identification techniques almost always involves measurement of the modal parameters of the structure. This relies on the fundamental assumption of linearity and superposition, see for example [1]. For nonlinear systems, a range of identification techniques have been proposed, many of which are discussed in the overviews by Worden and Tomlinson [2] and Kerschen et al. [3]. Approaches that have been applied to multi-degree of freedom systems using forced response data include the restoring force method [4,5], reverse path approaches [6] and NARMAX methods [7]. Bayesian fitting approaches are also being developed, although currently these mainly consider single degree-of-freedom systems [8,9]. An alternative to forced response data is to measure the resonant decay; this is done by forcing the system onto a resonance and then letting the response decay as presented in [10]. Feldman also developed an estimation method for single-degree of freedom systems [11] which has been applied to multi-degree of freedom systems with non-proportional damping [12] and a system with a localised nonlinearity [13].

To better understand the nonlinear dynamics of a structure, Rosenberg [14] discussed the idea of the nonlinear normal mode (NNM) as an extension to linear modes for nonlinear systems. Such modes were defined as a vibration-in-unison of the unforced, undamped system with all the degrees-of-freedom passing through zero and reaching their extreme values in unison with each other. Due to nonlinearities, NNMs often vary both as a function of maximum amplitude of vibration and

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over each oscillation of a mode. Notable early contributions to the development of NNMs includes Rand [15] and Vakakis et al. [16]. In addition, Shaw and Pierre [17] considered the presence of damping on such modes. Recently, NNMs have more typically been thought of as a periodic response of the unforced, undamped system, allowing, for example, responses in anti-phase such as whirling of a cable to be treated as an NNM [18].

NNMs can be shown in the frequency–amplitude projection revealing the backbones of the system. If these are considered in the linear modal domain, then the interactions between the linear modes that make up the various NNM solutions are revealed [19]. If the nonlinearities are *smooth*, analytical expressions have been derived for NNMs using perturbation techniques. Using normal forms [20,21], a state-space formulation for NNM responses has been reported [22,23]. More recently a normal form method that is applied directly to the modal equations of motion, the second-order normal forms [24,25], has been used to examine NNMs in terms of the interactions between modes of the linearised structure [18,19].

A methodology to extract the energy dependency of NNM modal curves and their frequencies from decaying time series is presented in [26]. It uses an extension of the force appropriation method [5] to nonlinear system and wavelet transform to perform the time-frequency analysis. This methodology to extract NNMs is later demonstrated experimentally in [27] using a cantilever beam with a geometric nonlinearity. They report the use of the conditioned reverse path method to identify the nonlinear behaviour of the beam. Following this, the identified model is used to produce theoretical NNMs which are compared to the experimentally measured NNMs.

Here we consider the use of experimentally measured resonant decay data, along with modal expressions for the equations of motion, as a means to identify the nonlinear parameters in a system. Unlike the approach demonstrated in [26,27], we use a methodology based on Moving Average filtering and zero-crossing detection to estimate the instantaneous frequency and amplitude of the decaying signal, thus extracting backbones curves in terms of amplitude–frequency maps as discussed in [28]. We use these amplitude–frequency relationships to identify nonlinear terms directly from the backbone curves since they can be more easily mapped onto stiffness nonlinearities. In this work, resonant decay data is taken from a nonlinear structure, a wing-nonlinear pylon inspired testpiece, and the experimental backbone curves of the test structure are estimated. Analytical expressions for the backbone curves are derived based on assumed nonlinear stiffness characteristics. These, together with the resonant decay data, are used to find suitable values for the nonlinear parameter in the model. The identification approach presented is applicable to lightly damped structural systems with smooth stiffness nonlinearities and resonant responses that are primarily dominated by their principal harmonic. This paper is organised as follows. Section 2 introduces the approach proposed in this work for the identification of structures containing elements with nonlinear stiffness. A simulated multi-degree-of-freedom (MDOF) system with two nonlinear elements is used in Section 3 to illustrate how the nonlinear parameters can be identified from the system backbone curves. The procedure is then applied to a nonlinear test structure in Section 4, closing with some final comments and remarks in the conclusions.

2. Identification of nonlinear parameters from experimental backbone curves

It is common for structures to be approximately linear at low vibration levels, but exhibit nonlinear behaviour as excitation levels increase. For this type of structure, the first task in system identification is to use conventional modal testing techniques, such as those described in [1], to identify the underlying linear system; that is, to estimate the linear mode shapes, natural frequencies and damping ratios within the frequency bandwidth of interest. Perhaps the fastest procedure consists of gently vibrating the test structure using broadband random excitation to measure the system's frequency response functions (FRF), and from them, to determine the modal model of the underlying linear system. Following this linear identification, the contribution of the nonlinear elements can be investigated.

A useful tool capable of offering an understanding of the behaviour of nonlinear systems is the backbone curve [19]. These curves define natural frequencies as a function of response amplitude for a system when no damping or forcing is present. A technique for extracting backbones involves estimating both the instantaneous amplitude and frequency of the nonlinear system over its free vibration response. Note that as the system will inevitably have damping, the response is only a good approximation to its undamped backbone curve if the damping is reasonably light.

In this work we use the methodology presented in [28] to estimate the system backbones. In brief, it consists of applying a force pattern to the structure at the relevant frequency of interest using harmonic excitation. Once the structure is responding at the desired resonance condition, the input is removed and the response of the structure is free to decay from the steady-state resonant response. The resonance decay response is then mapped into the linear modal space and the instantaneous frequency and amplitude envelope in terms of each linear modal coordinate calculated. As in [28], here we use zero-crossings and peak amplitudes to calculate the instantaneous frequency and amplitude envelop over time respectively.

As long as the level of vibration in the steady-state is large enough to activate the structural nonlinearities, the estimated backbone curves can be used to find various nonlinear parameters able to model the system dynamics. We recall that other methods such as those presented in [11,27] could be used to estimate backbone curves provided that the frequency–amplitude dependency in terms of modal coordinates are given.

To develop the nonlinear model it is necessary to hypothesise a nonlinear stiffness function that captures the form of the nonlinearity. Hints on this might be drawn from the form of the estimated backbone curves or from static test data if