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Parametric system identification of resonant micro/nanosystems operating in a nonlinear response regime

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ABSTRACT

The parametric system identification of macroscale resonators operating in a nonlinear response regime can be a challenging research problem, but at the micro- and nanoscales, experimental constraints add additional complexities. For example, due to the small and noisy signals micro/nanoresonators produce, a lock-in amplifier is commonly used to characterize the amplitude and phase responses of the systems. While the lock-in enables detection, it also prohibits the use of established time-domain, multi-harmonic, and frequency-domain methods, which rely upon time-domain measurements. As such, the only methods that can be used for parametric system identification are those based on fitting experimental data to an approximate solution, typically derived via perturbation methods and/or Galerkin methods, of a reduced-order model. Thus, one could view the parametric system identification of micro/nanosystems operating in a nonlinear response regime as the amalgamation of four coupled sub-problems: nonparametric system identification, or proper experimental design and data acquisition; the generation of physically consistent reduced-order models; the calculation of accurate approximate responses; and the application of nonlinear least-squares parameter estimation. This work is focused on the theoretical foundations that underpin each of these sub-problems, as the methods used to address one sub-problem can strongly influence the results of another. To provide context, an electromagnetically transduced microresonator is used as an example. This example provides a concrete reference for the presented findings and conclusions.

1. Introduction

There are many established uses of resonant microelectromechanical systems (MEMS) in inertial and pressure sensing applications [1,2], as well as emerging applications such as mass sensing [3], filtering [4] and timing [5], which are enabled by the small size and low power consumption metrics associated with these systems. As these devices continue to shrink to the nanoscale, the aforementioned advantages are often double-edged in that system characterization becomes more challenging. For example, the dynamic range, or the range of excitations where the response is linear, decreases with a reduction in scale [6], which inhibits the use of established linear characterization techniques.

System identification in the presence of nonlinearity can be a challenging problem. Due to the relevance of this problem to systems of many different scales, it has spawned a vast amount of research literature, of which a review can be found in

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[7]. Unfortunately, due to experimental constraints, the majority of system identification methods that have been developed cannot be used with micro/nanosystems, as a lock-in amplifier is used to acquire the typically small and noisy response of these systems [8–11]. This largely inhibits the use of time-domain methods, multi-harmonic methods, or frequency-domain methods, which rely upon a time-domain measurement, as the lock-in amplifier is only capable of measuring the amplitude and phase of a single harmonic of the steady-state response. The methods that are amenable to use when a lock-in amplifier is employed in the measurement system are based on fitting experimental data to an approximate solution of a reduced-order model. These approximate solution-based methods have applicability beyond micro/nanosystem characterization, as they have also been applied in the parametric identification of macroscale systems [12–16]. More recent works have adapted approximate solution-based methods for resonant micro/nanosystems [11,17–22], but there are significant limitations. This work is focused on these limitations, and more generally the conditions under which a resonant micro/nanosystem can be characterized by approximate solution-based methods.

In order to facilitate the parametric system identification of a resonant micro/nanosystem operating in a nonlinear response regime when a lock-in amplifier is employed, several sub-problems must be addressed: nonparametric system identification, or experimental design and data acquisition; the generation of physically consistent reduced-order models; the calculation of accurate approximate responses; and the application of nonlinear least-squares parameter estimation methods. This work is focused on the theoretical foundations of these sub-problems, and how each is intrinsically coupled to the others. Accordingly, a general theoretical framework is presented that is independent of the final application of the estimated parameters. Thus, while one could potentially the relax constraints that one of the sub-problems might introduce based on the final application, a common framework allows one to communicate with a larger audience, enables technical advancements in one application to benefit all of them, and mitigates the need for subjective expert knowledge.

Since this work is theoretical in nature, the only nonparametric system identification issue considered herein is noise. As will be shown in later sections, with a sufficiently accurate model, noise only influences the variability of the parameter estimates. Accordingly, the bulk of this work is focused on the latter of the three sub-problems noted above. It is important to point out that few prior works have, as identified by the authors, considered all four major aspects of parametrically identifying resonant systems operating in a nonlinear response regime. A notable exception, however, that focused on macroscale systems was [23]. In that work, a multi-term harmonic balance solution was used to produce steady-state responses for a model that included often-ignored, higher-than-third-order nonlinearities. Two major challenges that were also encountered by the authors of this work were issues related to converging to the correct solution, in particular when multiple stable solutions were present, and the sensitivity of the fully nonlinear estimation technique to initial parameter estimates. The authors of [23] elected to use a sub-optimal cost function as the fully nonlinear estimation technique was sensitive to initial parameter estimates. A significant limitation with using a sub-optimal estimation technique is that it does not necessarily satisfy the requirements needed for statistical inference, such as generating confidence intervals for the parameter estimates, performing hypothesis testing to compare the estimated model to the nominal one or a lower-order version, or ascertaining model adequacy.

While by no means the only way of modeling resonant micro/nanosystems, reduced-order methods are often preferable over others as they can accurately describe dynamic responses in a manner that can be used with the other sub-problems. Without considering the other sub-problems, though, this notion of accuracy can be subjective. There are many ways that the accuracy of a reduced-order model can be improved, such as the inclusion of multiple modes, but in this work, it will be shown that differences between third-order and fifth-order models at excitations close to the critical one, or the one at which bistability is present in the steady-state response, can be significant enough to prohibit accurate parametric identification. A common method of calculating approximate steady-state responses of a micro/nanoresonator is to use first-order perturbation methods. These methods are very useful, particularly in regards to design, as they can distill complex phenomena to something much simpler, but they can also introduce two issues: errors between the first-order solution and higher-order or more accurate solutions and effective parameter coupling.

There are several ways of estimating system parameters based on observed responses, such as maximum-likelihood methods, but the most commonly used for micro/nanosystem characterization are based on least-squares methods. In these methods, parameter estimation results from minimizing an error function that is equal to the squared sum of the residuals between the model and data. A fundamental assumption is that in the absence of noise, the nominal parameters yield an error function equal to zero. If a first-order perturbation-based solution, or more generally a deficient model, violates this zero error function condition, the estimated parameters may not correspond to the nominal parameters (i.e. the physical significance of the estimated parameters may be dubious). It is important to note that the use of a high-fidelity model introduces its own problems, such as the classic bias-variance issue [24]. However, since careful experimental design in conjuction with the proper use of a lock-in amplifier can significantly reduce the variability of the measured response, it is possible to estimate parameters that correspond to a high-fidelity model to within a reasonable tolerance. In addition, in order for the nominal parameters to minimize the error function, the error function's Hessian, evaluated with the nominal parameters, must be positive definite. If the first-order perturbation-based solution effectively couples parameters together, this positive definite condition is violated. Note that this issue of effectively coupled parameters is not limited to perturbation-based solutions, and can occur with any solution when its corresponding error function's Hessian is not positive definite.

The starting point for this research related to the parametric system identification of resonators operating in a nonlinear response regime arose from characterizing microscale cantilevers. Accordingly, Section 2 presents a fifth-order model for an

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