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Robust finite-time tracking control for nonlinear suspension systems via disturbance compensation

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ABSTRACT

This paper focuses on the finite-time tracking control with external disturbance for active suspension systems. In order to compensate unknown disturbance efficiently, a disturbance compensator with finite-time convergence property is studied. By analyzing the discontinuous phenomenon of classical disturbance compensation techniques, this study presents a simple approach to construct a continuous compensator satisfying the finite-time disturbance rejection performance. According to the finite-time separation principle, the design procedures of the nominal controller for the suspension system without disturbance and the disturbance compensator can be implemented in a completely independent manner. Therefore, the overall control law for the closed-loop system is continuous, which offers some distinct advantages over the existing discontinuous ones. From the perspective of practical implementation, the continuous controller can avoid effectively the unexpected chattering in active suspension control. Comparative experimental results are presented and discussed to illustrate the advantage and effectiveness of the proposed control strategy.

1. Introduction

Vehicle suspension systems have attracted great attention in automotive industry since they perform significant influence on ride comfort, vehicle stability and road handling [1–3]. Compared with the conventional vehicle suspension systems, active suspension systems can be more effective in isolating vibration between the frame of the vehicle and the irregular road surface. During the past decades, active suspension systems in research and development of modern vehicles have been extensively studied in top automakers, satisfying the requirements of enhancing comfort and improving handling. For example, Merceds-Benz employs an active body control system [1], which consists of passive suspension components and an actuator. Certainly, suspension systems still need increasing improvement of all aspects in both theoretical study and practical application. Various control strategies have been proposed for active suspensions to enhance suspension performance in recent years, ranging from linear control techniques [4–8] to nonlinear ones [9–13].

According to the existing control strategies for active suspension systems, the majority of them can ensure asymptotic stability or robust stability rather than finite-time stability. The effect of the asymptotic convergence rate is not ideal in many practical applications [14], and a reasonable finite-time stability is actually an urgent issue to be solved. Besides finite-time reaching an equilibrium, finite-time stability of a dynamic system might achieve better robustness performance, together with rapid dynamic and

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high-accuracy response. This is an excellent feature, such that the finite-time control technique is more concerned due to its practicability and realistic meanings [12]. Engineer also pay attention to the finite-time control problem since the settling time could be ensured for the system states to reach the equilibrium points. Under an uniqueness assumption, the authors in [15] reveal some significant regularity properties of setting time functions at the origin. Some sufficient and necessary conditions of finite-time stability are also considered for continuous systems in [16,17] and for discontinuous systems in [18]. In addition, numerous research results show that when a control plant is (locally) asymptotically stabile, there exists a negative degree of homogeneity, the system could achieve finite-time stability. This is an important reason for the homogeneity playing a vital part in finite-time controller design [19–26].

In active suspensions, the system model be derived as multi double-integrator chain form. Although the model seems simple, it is still an important model in control theory, due to the fact that this model can reflect many physical electrical and mechanical systems [27–29]. Moreover, a controller design approach proposed to the system of double integrator form could often be generalized for more general cases, which are always obtained by backstepping techniques. For the control plant of double-integrator chain form, many fundamental control strategies have been explored [15,19,20]. However, one important drawback of these state feedback controls is its limited capacity to handle the uncertainty of the control plant and external disturbance since it is difficult to establish an accurate model for physical systems in general conditions [30,31]. Thus, a robust technique is needed to deal with system uncertainties. For perturbed nonlinear systems, a disturbance compensator is usually very complicated to design, but it is the key factor to achieve the system robust capability. Thus, the disturbance compensation based finite time control scheme is needed to be further investigated.

As a direct and efficient way of disturbance rejection, the disturbance compensation-based finite time control scheme should be explored for a perturbed nonlinear system to effectively suppress disturbances. During the past decades, many disturbance rejection methods have been proposed for perturbed nonlinear systems [32–35], especially the sliding mode technique. The sliding mode controller has been well recognized for improving the robustness and satisfactory performance in many control systems. With a twisting control method in [36], a finite time control law is obtained for a double integrator system with external disturbance. However, the presented control law essentially is discontinuous and there exists chattering phenomenon, which is undesirable in practice. Using a low-pass filter technique [12,37], a chattering free full-order sliding mode is developed to realize continuous control. In addition, among various sliding mode algorithms, super twisting can be more promising as a potential methodology [23–26]. Through this method, the designed control law can be continuous and effectively avoid chattering, together with guaranteing all the major characteristics that exists in traditional sliding mode control system in the presence of bounded smooth disturbance [23–26,38,39].

Motivated by the aforementioned discussion [12,23–26,40,41], this study presents a novel finite-time tracking controller design for active suspension system with external disturbance. The theoretical results show that this is a simple and effective method to control suspension systems with external unknown disturbance. A super twisting-based continuous disturbance compensator is constructed to estimate the external disturbance in a finite time. This continuous compensator can overcome the disadvantages of chattering phenomenon caused by the high frequency switched inputs. Based on the finite-time separation principle, the homogeneous continuous nominal control law for the nominal system without disturbance can be designed separately from the compensator, which provides more flexibility in controller design and also a much improved performance. Thus, the overall control input is continuous since both of the nominal control and the compensator are continuous. The proposed continuous controller can guarantee the tracking error with respect to the desired reference trajectory in a finite-time. From the practical perspective, the finite-time continuous controller is more preferable and useful. The control performance is studied through comparative experiments using a prototype active suspension. These experimental results illustrate that the proposed control method has a very good robust performance and disturbance attenuation.

Notations. $\mathbb{R}$ represents the set of real numbers, $\mathbb{R}_+$ and $\mathbb{R}^n$ denote the set of nonnegative real numbers and n-dimensional real space, respectively.

2. System description and problem formulation

A quarter suspension model constituted by passive suspension components and an actuator is shown in the Fig. 1, which is a two degree-of-freedom system and widely used in the literature [11,12]. In this model, the sprung mass m_s and the unsprung mass m_u are connected by the nonlinear stiffening spring F_s, the nonlinear damper F_d and the active actuator u. The tyre can be modeled as a linear spring F_t with a damper F_b. Vertical displacements to the sprung, unsprung mass and road input are z_s, z_u and z_r, respectively.

The mathematical equations for the output forces of the suspension components and tyre are presented as follows:

$$F_s(z_s, z_u) = k_s(z_s - z_u) + k_n(z_s - z_u)^3 \tag{1}$$

$$F_d(\dot{z}_s, \dot{z}_u) = \begin{cases} b_e(\dot{z}_s - \dot{z}_u), \ \dot{z}_s - \dot{z}_u > 0 \\ b_c(\dot{z}_s - \dot{z}_u), \ \dot{z}_s - \dot{z}_u \le 0 \end{cases} \tag{2}$$

$$F_t(z_u, z_r) = k_f(z_u - z_r) \tag{3}$$

$$F_b(\dot{z}_u, \dot{z}_r) = b_f(\dot{z}_u - \dot{z}_r) \tag{4}$$