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Stochastic finite element model calibration based on frequency responses and bootstrap sampling

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A B S T R A C T

A new stochastic finite element model calibration framework for estimation of the uncertainty in model parameters and predictions from the measured frequency responses is proposed in this paper. It combines the principles of bootstrapping with the technique of FE model calibration with damping equalization. The challenge for the calibration problem is to find an initial estimate of the parameters that is reasonably close to the global minimum of the deviation between model predictions and measurement data. The idea of model calibration with damping equalization is to formulate the calibration metric as the deviation between the logarithm of the frequency responses of FE model and a test data model found from measurement where the same level of modal damping is imposed on all modes. This formulation gives a smooth metric with a large radius of convergence to the global minimum. In this study, practical suggestions are made to improve the performance of this calibration procedure in dealing with noisy measurements. A dedicated frequency sampling strategy is suggested for measurement of frequency responses in order to improve the estimate of a test data model. The deviation metric at each frequency line is weighted using the signal-to-noise ratio of the measured frequency responses. The solution to the improved calibration procedure with damping equalization is viewed as a starting value for the optimization procedure used for uncertainty quantification. The experimental data is then resampled using the bootstrapping approach and the FE model calibration problem, initiating from the estimated starting value, is solved on each individual resampled dataset to produce uncertainty bounds on the model parameters and predictions. The proposed stochastic model calibration framework is demonstrated on a six degree-of-freedom spring-mass system prior to being applied to a general purpose satellite structure.

1. Introduction

1.1. Literature survey

The problem of Finite Element (FE) model calibration has received much attention over the years because of its wide range of application in fields such as complex structural design, structural control and health monitoring and reliability and risk assessment [1]. The usual goal of FE model calibration is to use experimental data from a structural or mechanical system to reconstruct its unknown physical properties which appear as parameters in its numerical model such that it makes more precise predictions of the

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system response to a prescribed, or random, excitation [2].

The deterministic FE model calibration consists of searching for the best parameter setting that minimizes the error between model predictions and test data. A great number of researchers (e.g., [3–9],) have considered the model calibration problem using frequency domain data. Two common frequency domain based objective functions are formed from the least-squares or log-least-squares deviations in the Frequency Response Functions (FRFs). Through analysis and numerical case studies, it has been shown that a log-least-squares objective function gives better convergence properties and exhibits more numerical stability than a least-squares objective function [10,11]. However, these objective functions are in general nonlinear in parameters, which can lead to convergence problems in the optimization routines employed, meaning that stability and even existence of the solution of the optimization problem cannot be guaranteed. Recently, Abrahamsson and Kammer [12] proposed a model calibration formulation, called FE model calibration with damping equalization, based on the log-least-squares deviation in the FRFs in conjunction with the damping equalization technique. By this technique, the same fraction of critical modal damping is imposed on all modes of both the test data model, estimated from measurement data and the finite element model. Abrahamsson and Kammer [12] numerically demonstrated that this calibration formulation gives a smooth deviation metric with a large radius of convergence to the global minimum. Nevertheless, making predictions based on a single calibration of the model parameters is not best practice since it does not reflect our confidence in the predictions. The credibility assessment of an FE model must combine three activities: (*a*) an assessment of the fidelity to test data, (*b*) an assessment of the quantitative uncertainty in the model parameters, and (*c*) an assessment of the model prediction accuracy [13,14].

To quantify uncertainty, the parameter estimation problem can be formulated probabilistically in two distinct ways based on either the Bayesian or the Frequentist paradigm. The Bayesian approach views probability as a measure of relative plausibility of different possibilities conditional on available information [2]. The well-known Bayes' Theorem transforms a prior PDF, expressing the prior belief about the relative degree of plausibility of a certain parameter value, into a posterior PDF. This transformation is performed through the likelihood function, which reflects the degree to which the FE model can explain the measured data. Bayesian inference gains its popularity since the joint posterior PDF of the parameters accounts both for the uncertainty in the measurement and FE model predictions and the uncertainty in the prior knowledge. However, high dimensional integrals arising in the Bayesian formulation makes the use of the posterior PDF for the purpose of uncertainty quantification challenging. In early studies [15], the posterior PDF has been asymptotically approximated by a Gaussian PDF centered at the Maximum A Posteriori (MAP) point, *i.e.*, the parameter setting maximizing the posterior PDF, and characterized by a covariance matrix equal to the negative Hessian of the log posterior PDF evaluated at the MAP point. In recent years, the focus has shifted from asymptotic approximations to using Markov Chain Monte Carlo methods which generate (correlated) samples from the posterior PDF [16–20]. The distribution of the obtained samples gives the statistical description of the uncertain parameters.

In contrast, the Frequentist approach assigns a probability measure to an inferred value of a parameter setting through the notion of repetition [21]. The Maximum Likelihood Estimator (MLE), a frequently used estimator in the Frequentist literature, gives the parameter setting maximizing the likelihood function [11]. In analogy to the MAP estimate, if the model is identifiable and certain regularity conditions are met [22], the distribution of the MLE can be asymptotically approximated by a Gaussian distribution which covariance matrix is the inverse of the Fisher Information Matrix (FIM) given by the negative Hessian of the log-likelihood [23]. Bootstrapping is an alternative approach to quantifying the uncertainty in the model parameters. The idea behind bootstrapping is to repeatedly sample random datasets from the observed data and repeat the parameter estimation routine for each individual bootstrap dataset [24,25]. Bootstrapping results in a number of samples from the model parameters distribution which encodes the degree of confidence in the parameter estimates.

As part of the credibility assessment of the FE model, it is of special interest to propagate the model parameter uncertainty through forward simulation to obtain uncertainty estimates of structural response predictions [26,27]. In the case of identifiable models, the most straightforward approach for parameter uncertainty propagation is to use the assumed asymptotic distribution (Gaussian) for the parameter uncertainty. Such an approach can be realized through the projection of the approximated covariance matrix of the parameter estimates onto the first order sensitivities of the predictions of concern [11]. A more general approach to prediction uncertainty analysis is to directly use samples of the distribution of the parameter estimates. In the Bayesian setting, the prediction uncertainty can be quantified by integrating the predictions over the posterior distribution of the model parameter [28]. The solution to this integral gives a robust predictive distribution for the response of interest. This integral, which is often a high-dimensional integration, can be approximated using Monte Carlo methods due to the availability of a number of samples from the posterior distribution [2]. In the Frequentist setting, bootstrapping is a well-developed approach to assessing the credibility of the model predictions. In particular, this method is of interest when the goal is to quantify the prediction error and its variability for model responses at which there exists no measured data [29]. To this end, several bootstrap rules such as the 0.632 bootstrap rule have been developed which basically keep track of how well a model evaluated at an individual sample from the distribution of the parameter estimates predicts the response of interest at data points that are not included in the associated bootstrap dataset [29].

It should be mentioned that there also exist functional expansion-based methods [30], *e.g.*, Karhunen-Loeve expansion and polynomial chaos expansion, and the most probable point-based methods [31], *e.g.*, first-order (FORM) and second-order (SORM) reliability methods, for propagation of the parameter uncertainty to obtain the uncertainty in the model predictions. Since these methods fall outside the scope of this paper, they are not discussed in further details.

1.2. Statement of problem and contributions

In this paper, we consider the case when data is given in the frequency domain. In other words, a linear structure is excited by a pure sinusoidal signal and the output has settled to a stationary sinusoidal signal. The complex value of the transfer function at the