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Dynamic condensation approach to calculation of structural responses and response sensitivities

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ABSTRACT

Structural responses and response sensitivities are widely used in the finite element model updating, damage identification and optimization design. Calculation of the responses and response sensitivities of a large-scale structure consumes considerable computation storage and is usually time-consuming. This paper proposes an improved dynamic condensation approach to calculate the structural responses and response sensitivities. The condensed vibration equation is achieved by a simplified iterative scheme. By selecting the DOFs associated with the concerned element to be master DOFs, the response sensitivity is rapidly calculated from the derivatives of the master stiffness and mass matrices. Since the condensed vibration equation has a much smaller size than the original vibration equation, the proposed method is quite efficient in calculating the structural responses and response sensitivities. Finally, applications of the proposed method to an eight-storey frame and a cantilever plate demonstrate its accuracy and efficiency in the calculation of structural responses and response sensitivities.

1. Introduction

The sensitivity method is probably the most successful approach of the finite element (FE) model updating of engineering structures based on vibration testing data [1,2]. In vibration-based model updating and damage identification, the FE model is iteratively modified to ensure its vibration properties reproduce the measured counterparts in an optimal manner. In the optimization process, the structural responses are used to construct the objective function. The response sensitivities, the first derivatives of the structural responses with respect to the designed parameters, indicate the searching direction [3–7]. To accurately describe the practical structures, the analytical model is usually represented by a large model, including a large number of elements, degrees of freedom (DOFs) and structural parameters. Calculation of the structural responses of a large structure and the response sensitivities with respect to numerous designed parameters usually requires considerable computation storage and is a time-consuming process [1–7].

Model condensation, as an efficient technique to reduce the computational loads of large structures, was first applied to large FE models for faster computation of the natural frequencies and mode shapes. Model condensation methods remove

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some DOFs (slave DOFs) of the original FE model and represent the discarded DOFs with the retained DOFs (master DOFs). Afterwards, the eigen-functions of the condensed model are solved to approximate those of the original structures [8,9]. The model condensation technique is advantageous for solving a variety of engineering and mechanical problems [8–25]. Since the number of the master DOFs is much less than the total number of DOFs of the full model, the model condensation technique is helpful to reduce the large-size model and thus saves the computational resources and time [8–12]. In addition, the model condensation technique can be more promising if it is combined with the substructuring methods. The model condensation is performed on the independent substructures and on the interface coordinates of the substructures to improve computational efficiency [13–25].

Guyan [26] first proposed the static condensation technique to calculate eigensolutions. Friswell et al. [27,28] developed a dynamic and iterated improved reduced system (IRS) using an iterative method to obtain eigensolutions. Xia and Lin [9] improved the IRS method and achieved much faster convergence of the IRS method. Choi et al. [20] presented an iterated improved reduced procedure and a substructuring scheme for both the undamped and nonclassically damped structures. Weng et al. [29] extended the model condensation technique to calculate the eigensensitivities using the iterated dynamic condensation algorithm. These methods provided an efficient solution for large-scale eigenvalue problems. Soheilifard [30] extended the Guyan condensation method to the damped structures with a hierarchical non-iterative reduction method. Lima et al. [31] addressed a time domain condensation strategy for the viscoelastic linear and nonlinear systems, in which the viscoelastic behaviour is modelled with a four parameter fractional derivative model. As the precision and efficiency of model condensation methods are closely related to the selection of master DOFs, Bouhaddi and Fillod [32] proposed an approach to the selection of the master DOFs of the Guyan condensation method. Jeong et al. [33] proposed a rational primary DOFs selection method for a damped system according to the energy distribution of a structure. A variety of condensation methods have been developed to calculate the structural responses, whereas the sensitivity analysis by the condensed model is rarely studied. As calculation of response sensitivity usually consumes the majority of computation time in model updating, an effective condensed model for efficient calculation of response sensitivities is valuable.

In this paper, the structural responses and response sensitivities are calculated based on an improved dynamic condensation algorithm. A dynamic transformation matrix, obtained using a simplified iterative scheme, is derived to relate the responses of the master DOFs to the slave DOFs. Next, the large-scale global vibration equation is reduced into a condensed one. By including the DOFs of the concerned element in the master DOFs, the response sensitivities are computed efficiently from the derivatives of the master stiffness and mass matrices. The proposed method for the calculation of structural responses and response sensitivities is illustrated by an eight-storey frame and a cantilever plate.

2. Dynamic condensation to structural responses

The vibration equation of a structure with N DOFs is expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(\tau) + \mathbf{C}\dot{\mathbf{x}}(\tau) + \mathbf{K}\mathbf{x}(\tau) = \mathbf{F}(\tau) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ represent the mass, damping and stiffness matrices, respectively. $\mathbf{F}(\tau)$ is the excitation of the structure at time step τ. The structure is assumed to exhibit Rayleigh damping as $\mathbf{C}=a_1\mathbf{M}+a_2\mathbf{K}$, where a_1 and a_2 are the Rayleigh damping coefficients. $\ddot{\mathbf{x}}(\tau)$, $\dot{\mathbf{x}}(\tau)$ and $\mathbf{x}(\tau)$ are the acceleration, velocity and displacement of the structure, respectively, at time step τ.

Dividing the total DOFs of a structure into n_m master DOFs and n_s slave DOFs, the vibration equation is divided into [8,9]

$$\begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{ms}^{\mathrm{T}} & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_m(\tau) \\ \ddot{\mathbf{x}}_s(\tau) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{mm} & \mathbf{C}_{ms} \\ \mathbf{C}_{ms}^{\mathrm{T}} & \mathbf{C}_{ss} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{x}}_m(\tau) \\ \dot{\mathbf{x}}_s(\tau) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{ms}^{\mathrm{T}} & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_m(\tau) \\ \mathbf{x}_s(\tau) \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_m(\tau) \\ \mathbf{F}_s(\tau) \end{Bmatrix} \tag{2}$$

where the subscript 'm' represents the master DOFs, the subscript 's' represents the slave DOFs, and $N=n_m+n_s$. The superscript 'T' represents the transposed matrix.

A transformation matrix $\mathbf{t}$ is employed to relate the master DOFs and slave DOFs, and it is invariant to time and excitation, as follows [34]:

$$\mathbf{x}(\tau) = \begin{bmatrix} \mathbf{x}_m(\tau) \\ \mathbf{x}_s(\tau) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_m(\tau) \\ \mathbf{t}\mathbf{x}_m(\tau) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_m \\ \mathbf{t} \end{bmatrix} \mathbf{x}_m(\tau) = \mathbf{T}\mathbf{x}_m(\tau) \tag{3}$$

where $\mathbf{I}_m$ is an identity matrix with the size of $n_m \times n_m$. $\mathbf{T} = \begin{bmatrix} \mathbf{I}_m \\ \mathbf{t} \end{bmatrix}$ has the size of $N \times n_m$, and $\mathbf{t}$ has the size of $n_s \times n_m$.

Substituting Eq. (3) into Eq. (1) and premultiplying $\mathbf{T}^{\mathrm{T}}$ gives

$$\mathbf{M}_R\ddot{\mathbf{x}}_m(\tau) + \mathbf{C}_R\dot{\mathbf{x}}_m(\tau) + \mathbf{K}_R\mathbf{x}_m(\tau) = \mathbf{F}_R(\tau) \tag{4}$$

where

$$\mathbf{M}_R = \mathbf{T}^{\mathrm{T}}\mathbf{M}\mathbf{T} = \left(\mathbf{M}_{mm} + \mathbf{M}_{ms}\mathbf{t}\right) + \mathbf{t}^{\mathrm{T}}\left(\mathbf{M}_{ms}^{\mathrm{T}} + \mathbf{M}_{ss}\mathbf{t}\right) \tag{5a}$$