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New coherence function with measurements of one sampling period

1. Introduction

Discrete frequency response functions (FRFs) of a dynamic system are usually obtained using response and excitation series that are experimentally measured, which can be used to determine response spectra of the system under certain excitation spectra and to extract modal parameters of the system [1–4]. Discrete impulse response functions (IRFs) can be calculated using a least-squares (LS) method and an iterative method that directly uses response and excitation series based on the discrete Duhamel's integral [5], and associated discrete FRFs can be calculated by applying the discrete Fourier transform (DFT) to the IRFs [2]. A more efficient way of calculating discrete FRFs, compared with the aforementioned methods, is to apply the DFT to both response and excitation series and calculate the FRFs using some formulations. In the DFT, a series to be transformed is virtually extended to have an infinite length and be periodic with a period equal to the length of the series [6]. Periodic extension in the DFT can lead to erroneous FRFs when extended response and excitation do not conform to the physics of the system. However, when excitation is in the form of impulses or burst signals and the duration of one sampling period is so long that associated response can decay to zero at the end of one sampling period, resulting FRFs can be accurately measured with the application of the DFT and error in FRFs caused by periodic extension vanish.

The conventional coherence function has been proposed to evaluate qualities of discrete FRFs at each measured frequency [1,2], where DFTs of response and excitation series are used, and it has been widely used in experimental modal analysis [7–14]. A coherence function that has the same form of the conventional coherence function has been proposed to identify cutting tool wear and chatter, where bending vibration measured by two crossed accelerometers are used, and values of the coherence function reaching one at chatter and first natural frequencies indicate an onset of chatter and a severe wear stage of a cutting tool, respectively [15]. Response and excitation series of at least two sampling periods are needed to yield meaningful coherence function values, and stable function values often require response and excitation series of more sampling periods. A possible error in a data acquisition system (DAS) is response advancement error between measured response and excitation series. Response and excitation series measured by the DAS violate causality of a mechanical system. Response advancement error does not change the amplitude but the phase of a FRF measured by the DAS, compared with that of a FRF measured by a DAS without the error. However, response advancement error can never be reflected in the conventional coherence function due to its definition.

In this work, a new coherence function in Ref. [5], which is similar to those in Refs. [16,17], is introduced to efficiently and physically evaluate qualities of FRFs: meaningful and stable function values can be obtained when response and excitation series of only one sampling period are available, and errors in the FRFs and associated IRFs caused by system errors, such as response advancement error between response and excitation series in a DAS, can be reflected and quantified in the frequency domain, which constitutes novelty of this work. A fitting index in Ref. [5] is used based on the new coherence function to evaluate overall qualities of the FRFs and IRFs.

2. FRFs and coherence functions

Suppose an under-damped single-input-single-output system is under general excitation $f(t)$ with zero initial conditions. The excitation $f(t)$ to and response $y(t)$ of the system are discretely measured by a DAS with a sampling interval Δt, and an excitation series $f_i = f(i\Delta t)$, where $i = 1, 2, \ldots, n$, and a response series $y_j = y(j\Delta t)$, where $j = 1, 2, \ldots, 2n-1$, are available; n values of the IRF $h(t)$, i.e., $h_i = h[(i-1)\Delta t]$, where $i = 1, 2, \ldots, n$, can be calculated from the discrete Duhamel's integral:

$$[ft]_{(2n-1)\times n}[ht]_{n\times 1} = \left(\frac{1}{\Delta t}\right)[yt]_{(2n-1)\times 1} \tag{1}$$

where

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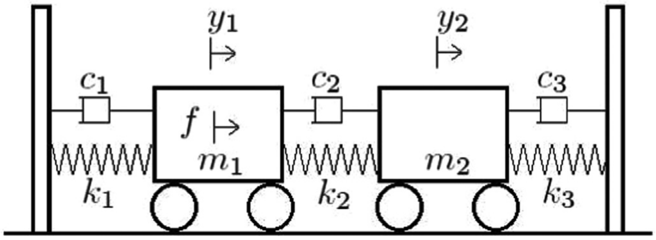

Fig. 1. A two-degree-of-freedom mass-spring-damper system.

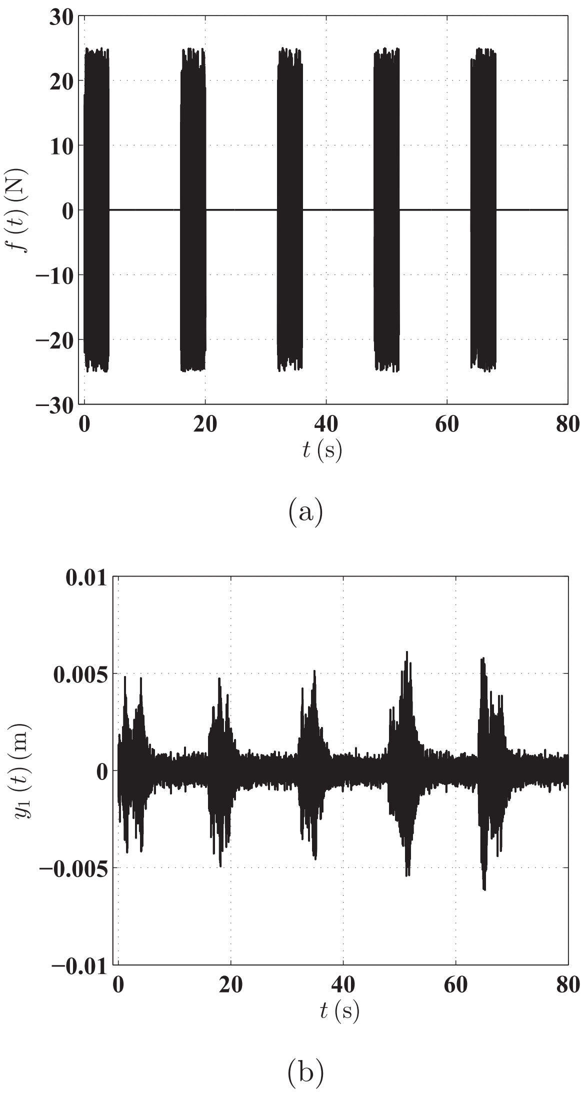

Fig. 2. (a) Burst random excitation and (b) the response of m_1 in Fig. 1 of five sampling periods.