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Superconvergent isogeometric free vibration analysis of Euler–Bernoulli beams and Kirchhoff plates with new higher order mass matrices

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Highlights

- We propose isogeometric higher order mass (HOM) matrices of thin beams and plates.
- The proposed method enables superconvergent free vibration analysis.
- The frequency accuracy is elevated by two orders using HOM matrices.
- The beam HOM matrices exhibit superior frequency accuracy and smaller bandwidth.
- The plate HOM matrices ensure superconvergent computation of arbitrary frequency.

Abstract

A superconvergent isogeometric free vibration analysis is presented for Euler–Bernoulli beams and Kirchhoff plates. This method is featured by new higher order mass matrices. For the 1D Euler–Bernoulli beam problem, it is shown that a new higher order mass matrix can be directly established by optimizing a reduced bandwidth mass matrix. The reduced bandwidth mass matrix is designed based upon the consistent mass matrix and it contains adjustable parameters to be determined via maximizing the order of accuracy of the vibration frequency. As a result, 4th and 6th orders of accuracy are observed for the proposed quadratic and cubic higher order mass matrices, while their corresponding consistent mass matrices are 2nd and 4th order accurate, respectively. Thus the higher order beam mass matrices have more superior frequency accuracy simultaneously with smaller bandwidth compared with their corresponding consistent mass matrices. While for the 2D Kirchhoff plate problem, in order to compute arbitrary frequency in a superconvergent fashion, a mixed mass matrix is formulated through a linear combination of the consistent mass matrix and the reduced bandwidth mass matrix. Then by introducing the wave propagation angle, the higher order plate mass matrix can be rationally derived from the mixed mass matrix. It is proved that the optimal combination parameter for higher order mass matrix depends on the wave propagation angle. Consequently a particular higher order mass matrix can always be set up for a superconvergent computation of arbitrary vibration frequency. It is shown that the quadratic and cubic higher order mass matrices possess 4th and 6th orders of accuracy, which are two orders higher than those of the consistent plate mass matrices. The construction of higher order mass matrices and the analytical results for vibration frequencies are systematically demonstrated by a set of numerical examples.

Keywords: Isogeometric analysis; Euler–Bernoulli beam; Kirchhoff plate; Free vibration; Higher order mass matrix; Superconvergence

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1. Introduction

The isogeometric analysis introduced by Hughes et al. [1] directly passes the model information in the computer aided geometry design (CAGD) as the input to the subsequent finite element analysis (FEA). The same basis functions, i.e., the B-splines or the non-uniform rational B-splines (NURBS), are used for the geometric representation in CAGD as well as the field variable approximation in FEA. Thus the conventional gap between CAGD and FEA is completely removed since no additional meshing effort is required for FEA. The geometry exactness is maintained throughout the isogeometric analysis. At the same time, the isogeometric analysis enjoys the higher order smoothing approximation properties of the basis functions, as is especially desirable for the problems with higher order governing differential equations, such as Euler–Bernoulli beam and Kirchhoff plate problems. During the past decade this method has experienced very rapid evolutions and fast growing applications with superior performance, for example, structural vibrations and wave propagations [2–6], structural optimizations [7–12], plates and shells [13–19], phase field modeling [20–22], contact problems [23–26], fracture and damage problems [11,22,27,28], fluid structure interactions [29–32], large eddy simulations of turbulence [33–36], among others. Noticeable efforts were also spent on the coupling of isogeometric analysis with other numerical techniques to fully employ the advantages of different approaches, i.e., immersed isogeometric boundary methods [37–39], isogeometric boundary element methods [40–45], and coupled isogeometric-meshfree methods [46,47], etc. Recently, Schillinger et al. [48] presented reduced Bezier element quadrature rules for isogeometric analysis which significantly enhance the computational efficiency.

As for the structural vibration analysis, it has been thoroughly shown by Cottrell et al. [2,3], Reali [4], and Hughes et al. [5] that the smoothing positive B-spline or NURBS basis functions in isogeometric analysis yield much more accurate frequency spectra than the conventional equal order C^0 finite element approximations. Subsequently quite a few of works have been devoted to the vibration problems. The free vibration of thin plates with isogeometric analysis was carried out by Shojaee et al. [49]. Thai et al. [50] performed free vibration and buckling analysis of laminated composite shear deformable plates via the isogeometric approach. Auricchio et al. [51,52] introduced an isogeometric collocation method for explicit dynamics. The vibration of functionally graded structures using NURBS-based finite elements was studied by Valizadeh et al. [53] and Taheri et al. [54]. Weeger et al. [55] employed the isogeometric method for nonlinear vibration analysis of Euler–Bernoulli beams. Meanwhile, Lee and Park [56] analyzed the vibration of Timoshenko beams based upon the isogeometric approach. An isogeometric analysis framework for the rotation-free bending-stabilized cable, bending strips of multiple cables and cable–shell coupling was developed by Raknes et al. [57]. A comprehensive comparison of the finite element and NURBS approximations for general eigenvalue, boundary-value, and initial-value problems was recently presented by Hughes et al. [58]. Reali and Gomez [59] proposed an isogeometric collocation approach for Bernoulli–Euler beams and Kirchhoff plates. These developments uniformly demonstrated that the isogeometric analysis gives excellent frequency results for structural vibrations. However, the nice performance of isogeometric structural vibration analysis is tied with the employment of consistent mass matrices [2–4]. On the other hand, the lumped mass matrices do not yield satisfactory solution accuracy, which even cannot be improved by elevating the order of the basis functions [2]. For example, 2nd order frequency accuracy of Euler beams is observed for both the quadratic and cubic lumped mass matrices, while the corresponding quadratic and cubic consistent mass matrices produce 2nd and 4th orders of accuracy, respectively.

The design of superaccurate algorithms for structural vibration analysis is an interesting and important topic. In the context of finite element methods, several methods have been proposed to develop superaccurate formulations for vibration analysis [60–64]. Among these methods, one common way to improve the frequency accuracy is the employment of modified mass matrices, i.e., the higher order mass matrices [60,61,63,64]. Meanwhile, the discretization errors in the parametric mass and stiffness formulations can be effectively minimized by an inverse method [62]. In [63,64], it has been shown that the finite element higher order mass matrices can often be constructed as an optimal combination of the corresponding consistent and lumped mass matrices. Nonetheless, it was shown in [65] that this approach does not yield higher order mass matrices for isogeometric analysis. In order to improve the accuracy of isogeometric vibration analysis, the present authors [65] proposed a set of higher order mass matrices for 1D rod and 2D membrane problems. It was shown that compared with the consistent mass matrices, the higher order mass matrices can upgrade the free vibration frequency accuracy by 2 orders. The 1D rod and 2D membrane higher order mass matrices were both established with a two-step method [65]. In the first step, under the mass conservation constraint a reduced bandwidth mass matrix that has the same order of accuracy as the consistent mass