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Bifurcation tracking by Harmonic Balance Method for performance tuning of nonlinear dynamical systems

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ABSTRACT

The aim of this paper is to provide an efficient frequency-domain method for bifurcation analysis of nonlinear dynamical systems. The proposed method consists in directly tracking the bifurcation points when a system parameter such as the excitation or nonlinearity level is varied. To this end, a so-called extended system comprising the equation of motion and an additional equation characterizing the bifurcation of interest is solved by means of the Harmonic Balance Method coupled with an arc-length continuation technique. In particular, an original extended system for the detection and tracking of Neimark-Sacker (secondary Hopf) bifurcations is introduced. By applying the methodology to a nonlinear energy sink and to a rotor-stator rubbing system, it is shown that the bifurcation tracking can be used to efficiently compute the boundaries of stability and/or dynamical regimes, i.e., safe operating zones.

1. Introduction

Industrial requirements in terms of security, cost reduction and increased performance push designers, manufacturers and operators to create more and more advanced technological equipment in which nonlinearities are now common. In this context, understanding and controlling nonlinear effects due to contact, large deflections, links or components such as bearings or friction dampers is an important issue. Resulting nonlinear systems can exhibit complex dynamical behaviours with specific features such as multi-solutions for a single value of the system parameters, amplitude or frequency jumps, internal resonances, period-doubling, quasi-periodic or chaotic motions [1–4]. However, for a given system, the systematic study of all these phenomena and their possible occurrence is generally out of reach because of the large number of parameters to be considered and the limited available computational resources. An overall understanding of the system's dynamics can nevertheless be obtained through the computation of periodic solutions, forced response curves and associated bifurcations.

The literature comprises various numerical methods for the direct computation of periodic solutions which can be classified into two main categories, namely time domain and frequency domain approaches. The shooting method [5] and orthogonal collocation [6] which rely on solving a nonlinear boundary value problem are two popular time domain approaches. Orthogonal collocation is implemented for instance in AUTO [7] and MATCONT [8] softwares. In the frequency domain, the most commonly used method is certainly the harmonic balance method (HBM) which consists in approximating the unknown state variables by means of truncated Fourier series. Since nonlinearities cannot be directly computed in the frequency domain, the standard HBM is usually coupled with the alternating frequency-time (AFT) scheme [9] which computes the nonlinear terms in the time domain and subsequently their

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Fourier coefficients. The AFT scheme is very popular due to its easy implementation, its computational efficiency and its ability to handle almost any type of nonlinearities. Over the past decades, the HBM has been extended to quasi-periodic solutions [10–13] and many improvements have been proposed, such as adaptive schemes that improve the performance by selecting only the harmonics of interest [14,15] as well as methods to handle systems with many distinct states [16] and strong or non-smooth nonlinearities [17,18]. For the computation of forced response curves, i.e. the following of periodic solutions when a control parameter is varied, the HBM is coupled with a continuation technique, e.g. the arc-length continuation based on tangent prediction steps and orthogonal corrections [19,20] or the so-called asymptotic numerical method [21].

In an engineering context, the local stability of periodic solutions is often computed when following the response curve since it distinguishes between solutions that may or not be experimentally observed. Several algorithms operating either in the time or frequency domain are available [22,23]. The detection of bifurcations points is more rarely performed. However, their computation is of prime interest. For instance, a limit point (also called fold bifurcation) indicates a change of stability and is responsible for amplitude jumps that can lead to significant and possibly dramatic changes in the system response. A Neimark-Sacker (secondary Hopf) bifurcation corresponds to a change of motion regime and indicates the transition from a periodic to a quasiperiodic motion.

Consequently, the parametric analysis of bifurcations can be used to understand the effects of nonlinear phenomena and to determine the boundaries of stability and/or dynamical regimes, i.e., safe operating zones. The resulting bifurcation map is an efficient tool for designers in order to identify the relevant parameters ruling the system's behaviour and to choose appropriate sets of parameters that lead to optimal runs. A simplified approach for this parametric analysis consists in calculating the whole response curves for several values of a chosen parameter, and collect all the detected bifurcations. However, this approach is very expensive and produces unnecessary results since only bifurcation points are of interest. A more efficient approach consists in detecting a starting bifurcation point for a fixed value of the parameter of interest, then in directly tracking the path of bifurcations while this parameter is varied.

Two approaches exist for the precise computation of bifurcation points. The first one is based on the use of so-called standard extended systems and consists in introducing one or more additional equations characterizing the bifurcation. The second approach relies on minimally extended systems and bordering techniques in which only one scalar function is added. The direct calculation of limit points of nonlinear equations depending on a parameter was first introduced by Seydel [24,25], Moore and Spence [26] using standard extended systems. Many authors also utilized this approach for the direct calculation of critical points for post-buckling finite element problems [27–29]. It was recently combined with HBM by Petrov [30] for the detection of branch point bifurcations, where two branches of solutions intersect, and branch-switching along curves of periodic solutions. The direct calculation of limit points by means of minimally extended systems was introduced in [31] and subsequently used and improved by many authors [32,33]. The computation of Hopf bifurcations for dynamical systems by means of standard extended systems originates from the work of Jepson [34]. Several variations and improvements have then been developed by Griewank and Reddien [35] or Roose et al. [36,37] among others. This type of algorithm is frequently used in fluid mechanics to detect instabilities when the Reynolds number reaches critical values [38]. Such standard extended systems are implemented in AUTO [7] and LOCA [39] softwares. The computation of Hopf bifurcations by means of minimally extended systems is detailed in [32,40,41]. These minimally extended systems are implemented in MATCONT [8] software. A comprehensive review of the methods suitable for detecting bifurcations can be found in [19] while in [42] authors focus on Hopf bifurcations.

The numerical continuation of bifurcation points is much less addressed in the literature. The continuation of paths of limit points of nonlinear equations having two parameters was first investigated by Jepson and Spence [43] with standard extended systems. In a mechanical context, it was later used for studying the sensitivity of critical buckling loads to imperfections [44–46]. In MATCONT, the continuation of codimension-1 bifurcations of dynamical systems is performed by means of minimally extended systems. In [47], Detroux et al. combined this approach with the HBM for the tracking of limit point, branch point and Neimark-Sacker bifurcations of large-scale mechanical systems. In this paper, we combine HBM and standard extended systems. We already used this approach in [48] in the case of limit points. Here, this work is extended to all types of codimension-1 bifurcations. In particular, we build on the work of Griewank and Reddien [35] in order to propose an efficient algorithm for the computation and the tracking of Neimark-Sacker bifurcations.

The paper is organized as follows. The formulation of the harmonic balance method for the continuation of periodic solutions is presented in Section 2. The stability analysis and the characterization of the bifurcations are based on the Floquet exponents obtained from a quadratic eigenvalue problem as described in Section 3. The extended systems used for the computation of the bifurcations are then detailed in Section 4, with emphasis on computational issues such as the efficient calculation of the derivatives involved in the Newton-Raphson iterations. The direct tracking of these bifurcations, i.e., the continuation of bifurcation curves is addressed in Section 5. The performance of the proposed approach is demonstrated in Section 6 on two nonlinear dynamical problems: a nonlinear vibration absorber and a nonlinear Jeffcott rotor. Finally, conclusions are drawn in the last section.

2. Equilibrium path

2.1. Harmonic Balance Method

A forced nonlinear dynamical system with n degrees of freedom (DOFs) governed by the following set of equations of motion is considered

$$r(\boldsymbol{x}, \omega, t) = \boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) + \boldsymbol{f}_{nl}(\boldsymbol{x}, \dot{\boldsymbol{x}}) - \boldsymbol{p}(\omega, t) = \boldsymbol{0} \tag{1}$$