Mechanical Systems and Signal Processing ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Flap-wise and chord-wise vibrations of axially functionally graded tapered beams rotating around a hub

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ARTICLE INFO

Article history:
Received 15 April 2016
Received in revised form
3 June 2016
Accepted 11 July 2016

Keywords:
Centrifugally stiffened beams
Axially functionally graded beams
Flap-wise/chord-wise vibrations
Rayleigh–Ritz method
Euler–Bernoulli theory

ABSTRACT

This paper presents flap-wise and chord-wise flexural vibration analyses for centrifugally stiffened tapered beams made of functionally graded material in axial direction. Functions of material properties varying along beam are defined in terms of the power law distribution. Calculations are conducted by simple computation technique of the Rayleigh–Ritz method that uses simple shape functions and energy expressions written for centrifugally stiffened Euler–Bernoulli beams. Effects of taper ratio, hub radius, angular velocity and non-homogeneity are inspected for the thin beams with several classical boundary conditions. Results given as non-dimensional natural frequencies are validated by the results given in existing literature and/or the outputs of finite element analyses performed for axially functionally graded solid beam. Achievements and limitations of the method are discussed and clearly reflected.

1. Introduction

This paper is on the flexural vibration of tapered and axially functionally graded (AFG) beams rotating around a hub. Beam type elements can rotate around a hub in many applications such as turbine blades, wind turbines, propellers and etc. Design of these components requires taking mechanic, dynamic, aerodynamic and thermal limitations into account to supply the most proper economic conditions and increase the energy efficiency. In order to do this, usage of functionally graded (FG) beams has attracted researchers' attention in recent years. FG components preserve structural integrity by providing appropriate power and weight distributions. Smooth change of material properties removes stress loads, which occur in layered composites, and accordingly eliminates delamination faults. A centrifugally stiffened FG beam may be broken due to dynamic and aerodynamic effects leading resonance although it has sufficient mechanic and thermal resistance. FG beams should have appropriate modal characteristics to avoid resonance. Changes in modal characteristics are easily followed by the natural frequency parameters. Therefore, calculation of natural frequencies plays significant role in achievement of novel designs for rotating FG beams.

Researchers have paid great attention to the flexural vibration of uniform or non-uniform beams rotating around a hub for three decades. Selected papers considering the flap-wise vibration of rotating uniform Euler–Bernoulli beam are presented by Banerjee, Chung and Yoo, Yang et al., Yoo and Shin [1–4]. Chord-wise vibration analysis applied only for uniform cantilever beams is introduced by Cheng et al. [5]. Beside this, vibration of rotating non-uniform Euler–Bernoulli beams is

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http://dx.doi.org/10.1016/j.ymssp.2016.07.017

Please cite this article as: K. Mazanoglu, S. Guler, Flap-wise and chord-wise vibrations of axially functionally graded tapered beams rotating around a hub, Mech. Syst. Signal Process. (2016), http://dx.doi.org/10.1016/j.ymssp.2016.07.017

analysed in several research works [1,6–9]. Rotating beam vibrations are also studied using the Timoshenko beam model [10–12]. Most of these works use the finite element based analyses to reach approximate solution [2,3,7,9]. Dynamic stiffness matrix method [1], differential transform method [8], Galerkin's method [5,10], and Rayleigh–Ritz method [4,11] also take place in other works of current literature.

Studies on vibration analysis of FG beams have tremendously increased in recent years. Vibration of non-rotating AFG beams is inspected in some papers [13–15]. Huang et al. [13] define an additional function to obtain solution using power series method. Huang and Li [14] present a simple procedure to find natural frequencies by the transformation of governing equation with varying coefficients to Fredholm integral equations. Shahba et al. [15] introduce a new element type to reach solution using finite element analysis. Transversely FG beams are also the subject of studies on vibration analyses which are carried out by the Rayleigh–Ritz method [16] and Navier solution procedure [17].

In current literature, there are a few studies on vibration of rotating beams made of FG materials. Ramesh and Rao [18] consider the vibration of transversely FG rotating beams using the Rayleigh–Ritz method. Flap-wise vibration analyses have been recently carried out for various classical boundary conditions of rotating AFG beams [19–22]. In order to do this, Zarrinzadeh et al. [19] apply a finite element based approach while Rajasekaran [20] demonstrates applicability of the differential transform and quadrature methods. Flap-wise vibration analyses of rotating AFG beams are also achieved using the Ritz method with simple [21] and Chebyshev polynomials [22].

To the best of author's knowledge, usage of the Rayleigh–Ritz method for the flap-wise and chord-wise vibrations of centrifugally stiffened AFG beams is firstly investigated in this paper. Comparative results for tapered AFG beams rotating around a hub are given after introducing theoretical background of the beam model and computation technique of the Rayleigh–Ritz solution method.

2. Theoretical background

In this work, cross-sectional properties of rotating beam are assumed to vary along the length, L, of beams as shown in Fig. 1. Cross-section changes of the tapered beams are expressed by following functions representing height, h, and width, b.

$$h(x) = h_0\left(1 - c_h\frac{x}{L}\right) \tag{1}$$

$$b(x) = b_0\left(1 - c_b\frac{x}{L}\right) \tag{2}$$

where h_0 and b_0 symbolise the dimensions at the root of beam. c_h and c_b are the taper ratios of the height and width respectively. Thus, cross-sectional area, $A(x)$, and area moment of inertias, $I_{yy}(x)$, $I_{zz}(x)$, are written as follow:

$$A(x) = b(x)h(x) \tag{3}$$

$$I_{yy}(x) = b(x)h(x)^3/12 \tag{4a}$$

$$I_{zz}(x) = h(x)b(x)^3/12 \tag{4b}$$

On the other hand, material parameter, p_m, which may be modulus of elasticity (E) or density (ρ), is assumed to vary as:

$$p_m(x) = (p_{m1} - p_{m2})\,(x/L)^n + p_{m2} \tag{5}$$

where p_{m1} and p_{m2} denote the parameters of two different materials. n is the material non-homogeneity factor. Within this framework, energy expressions for flap-wise and chord-wise vibrations of centrifugally stiffened AFG beams and details of a technique implemented for the solution method are presented in following subsections.

2.1. Energy expressions for centrifugally stiffened beams

In this paper, vibration of centrifugally stiffened beams is considered in both flap-wise and chord-wise directions as illustrated in Fig. 2. It is assumed that bending-torsion and bending-bending coupling effects are negligible for the beam selected.

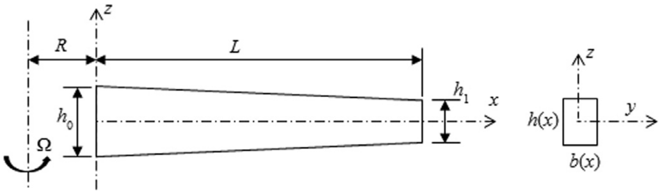

Fig. 1. Configuration of tapered rectangular beam rotating around a hub.

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