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A spectral dynamic stiffness method for free vibration analysis of plane elastodynamic problems

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ABSTRACT

A highly efficient and accurate analytical spectral dynamic stiffness (SDS) method for modal analysis of plane elastodynamic problems based on both plane stress and plane strain assumptions is presented in this paper. First, the general solution satisfying the governing differential equation exactly is derived by applying two types of one-dimensional modified Fourier series. Then the SDS matrix for an element is formulated symbolically using the general solution. The SDS matrices are assembled directly in a similar way to that of the finite element method, demonstrating the method's capability to model complex structures. Any arbitrary boundary conditions are represented accurately in the form of the modified Fourier series. The Wittrick-Williams algorithm is then used as the solution technique where the mode count problem (J_0) of a fully-clamped element is resolved. The proposed method gives highly accurate solutions with remarkable computational efficiency, covering low, medium and high frequency ranges. The method is applied to both plane stress and plane strain problems with simple as well as complex geometries. All results from the theory in this paper are accurate up to the last figures quoted to serve as benchmarks.

1. Introduction

A wide range of three-dimensional elastodynamic problems are generally treated by two-dimensional (plane) theories, which include plane stress and plane strain theories. The plane stress theory assumes that the stress perpendicular to the plane under consideration is always zero. This is often the case for plates whose upper and bottom surfaces are free. The vibration of such a plate in its own plane is generally called inplane vibration. Despite the fact that the transverse vibration [1] is usually given more importance for plate-like structures which are more easily excited by transverse external forces rather than inplane forces, there are many instances when inplane vibration can have pronounce effects. As a consequence, there has been an increasing interest in the inplane vibration of plates and plate assemblies. For instance, inplane vibrations are very important for built-up structures [2] where two or more plates are connected at a certain angle such that the transverse and inplane vibrations are directly coupled. The inplane vibrations become even more important in the mid to high frequency ranges for noise control and energy transmission analyses of structures [3,4]. Examples include the walls of aerospace structures, the hulls of ships and cutting tools.

The plane strain theory on the other hand, is widely used to investigate the free vibration of engineering structures like earth dams [5], shear wall structures [6] and thin or thick hollow cylinders [7–10]. For example, the earth dams and shear wall structures are designed to counter the effect of lateral dynamic loads caused by earthquake or wind. The plane strain theory is also widely used

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in plane wave propagation problems [11], which have applications in non-destructive testing [12,13] and phononic crystal analysis [14]. Some other investigators have used the plane strain theory to study the mechanism of edge effects on the natural vibration and wave propagation properties of thick multi-layered plates [15]. As the natural mode shapes can be regarded as the standing waves of a structure with the prescribed boundary conditions, the plane strain free vibration can provide important information for wave propagation problems with respect to different boundary conditions or discontinuities.

Without doubt, the above problems can be solved by the finite element method (FEM) with many well-developed commercial packages which can handle complex geometries. However, the FEM may become inadequate and unreliable when modelling structures within medium to high frequency ranges. In order to capture the relatively short wavelengths of structural deformations in these frequencies, an FEM model may require prohibitively large number of degrees of freedom (DOF) and even then the results can be still unreliable. Furthermore, for optimisation and parametric studies, the FEM becomes less attractive because of the considerable computational cost and/or the requirement of remeshing the structures. Therefore, analytical methods that are both efficient and accurate should be developed, which will facilitate efficient parametric and optimisation studies by varying significant parameters.

There are a few exact or analytical methods for plane elastodynamic problems, but even so, these methods are generally limited to simple geometries and restricted boundary conditions. It is well known that the closed-form exact solution for free inplane vibration (plane stress) is available only for rectangular plates with a pair of opposite edges simply supported. The earliest research on this topic was probably conducted by Lord Rayleigh [16]. Much later, Gorman [17] carried out a thorough investigation for exact solutions of simply supported plates by using Levy-type solutions. Xing and Liu [18–20] provided closed-form exact solutions for all possible cases of simply supported plates by using the Rayleigh quotient method. The classical dynamic stiffness method [21–24], first developed for plates in the 1970s [21], can be applied to plate assemblies but restricted to cases with two opposite plate edges simply supported. Plates with other boundary conditions are solved resorting to other analytical methods. Bardell et al. [22] used the Rayleigh-Ritz method to discuss the free inplane vibration of single plates with simply supported, fully clamped and completely free boundary conditions. Dozio [25] used the Ritz method in conjunction with a set of trigonometric functions to study the free inplane vibration of plates with elastic boundaries. Farag and Pan [26,27] made use of two types of series solution in the forced response analysis to examine the inplane vibration of rectangular plates with a pair of opposite edges clamped and other two edges being either clamped or free. The same cases were solved by Wang and Wereley [28], utilising the Kantorovich variational method. Gorman employed a systematic superposition method to study the free inplane vibration of completely free [29] and fully clamped [17] plates. Nefovska-Danilovic et al. [30] developed the dynamic stiffness method for isotropic rectangular plates based on Gorman's superposition method. Du et al. [31,32] used a Fourier series based analytical method to examine the free inplane vibration of plates with different boundary conditions. More recently, Papkov [33] provided the lower and upper bounds of natural frequencies for the free inplane vibration of completely free and fully clamped plates by an analytical method which makes use of the asymptotic behaviour of quasi-regular infinite systems. There is much less work on the free vibration of 3D solid structures under plane strain deformation. Such analysis is generally based on numerical methods. Tsiatas and Gazetas [5] applied an FEM model for plane-strain free vibration of earth dams. Nardini and Brebbia [6] developed a boundary element method for plane strain vibrations. There are even less papers on analytical methods for plane strain vibration. Gazis [7] derived the exact solution for the plane-strain vibration of a thick hollow cylinder. Ahmed [8] used a generalised Fourier-series technique for the axisymmetric plane-strain vibrations of a thick-layered orthotropic cylinder. Dong and Goetschel [15] made use of a direct-iterative eigensolution technique to investigate the edge effects in laminated plates. However, most of the above analytical methods are limited to single rectangular or annular domain and thus can not be applied to complex geometries.

There is a recently developed analytical method called the spectral dynamic stiffness method (SDSM) [34–36] which can handle complex structural geometries with any arbitrary boundary conditions. This method has earlier been developed for the biharmonic equation [34–36] which governs the transverse vibration of thin plates. The formulated spectral dynamic stiffness (SDS) matrices can be assembled directly to allow modelling of complex geometries in a similar way to the FEM, but importantly the exploited shape function in the SDSM is exact as opposed to approximate as in the FEM. Therefore, highly accurate solutions can be obtained from the SDSM by using as few elements as possible. Besides, the SDS formulation represents infinite degrees of freedom (DOF) accurately and efficiently by using only a very few DOF along the structure boundaries. As a results, the proposed method can provide highly accurate natural frequencies and modal shapes with remarkable computational efficiency, which is much superior to both the conventional FEM and BEM, not only within low frequency range, but also within medium to high frequency ranges. Furthermore, the SDSM has the certainty that no natural frequency of the structure will be missed and no spurious modes will be captured. The above superiorities of SDSM plus its analytical essence provide a huge advantage for parametric studies and structural optimisation.

The main purpose of this paper is to extend and generalise the previous SDSM for biharmonic equation [34–36] to Navier's equation which governs plane elastodynamic problems covering both plane stress and plane strain assumptions. However, the SDSM development for plane elastodynamic problems in the current research is different and indeed a formidable challenge compared to that in the biharmonic equation for thin plates [35,36,34]. This is due to the fact that previous investigations [35,36,34] involved only one variable as opposed to two variables encountered here. Moreover, there is a 90° phase differences between the expressions for the two variables and between the associated boundary conditions in the plane elastodynamic problems. All of the above differences increase the complexity of the problem many folds, given the fact that completely arbitrary BCs will be accounted for and analytical instead of numerical formulations will be developed. Therefore, the earlier SDS symbolic formulation through the solution of the biharmonic equation [35,36,34] as well as the associated building blocks (e.g., modified Fourier series and the J_0 count problem) need to be generalised in the new SDSM development for plane elastodynamic problems. More importantly, the