ARTICLE IN PRESS

Mechanical Systems and Signal Processing xx (xxxx) xxxx–xxxx

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Order-frequency analysis of machine signals

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ARTICLE INFO

Keywords:
Order-frequency analysis
Angle/time cyclostationary
Rotating machines
Variable speed conditions
Order-frequency spectral correlation
Improved envelope spectrum

ABSTRACT

Latest researches have asserted the eligibility of angle/time cyclostationarity in analyzing machine signals when operating under variable speed conditions. A core descriptor in this framework is the order-frequency spectral correlation (OFSC), basically estimated by the *averaged cyclic periodogram* (ACP), being able to jointly decode (i) the angle-dependent modulations related to the machine kinematics and (ii) the time-dependent carriers related to the machine dynamics. The present paper comes into this context with the aim of enriching this framework with new tools excerpted from cyclostationarity. In particular, a new estimator of the OFSC based on the *cyclic modulation spectrum* (CMS) is proposed and compared with the ACP in terms of resolution, statistical performance and computational cost. In addition, two related tools are theoretically addressed and their estimators are derived through the ACP and CMS. Specifically, the optimality of the "order-frequency spectral coherence" (the normalized/whitened form of the OFSC) in revealing cyclic components according to their SNR is demonstrated. Also, the "improved envelope spectrum" is derived from the latter by integrating over the spectral frequency variable, evidencing considerable enhancement over the squared envelope spectrum. The potentiality of the proposed tools and the adequacy of the related estimators are experimentally investigated on simulated and real-world vibration signals.

1. Introduction

The theory of cyclostationary (CS) processes has proven its efficiency in many fields of science such as meteorology, biology, economy, telecommunication and mechanics [13]. Thanks to this theory, the last two decades have particularly witnessed spectacular progresses in signal processing of machine signals [20]. Specifically, many CS-based techniques have been proposed to deal with various mechanical problems such as machine diagnostics [12,58], system identification [6] and source separation [1–3]. In this context, cyclic spectral analysis is a core discipline of cyclostationarity. It particularly concerns (second-order) CS signals and offers a set of useful tools to analyze them [23]. Being based on the covariance function, the robustness of these tools follows from the consideration of the information redundancy across the cycles (as opposed to the approaches based on local stationarity), thus making it suitable to reveal weak cyclic signatures even when embedded in strong stationary noise [11].

The *spectral correlation* (SC)— originally proposed by Gardner in Ref. [8]— is one of the most efficient tools to detect CS patterns in stochastic signals. It is defined as the double Fourier transform of the covariance function. It is therefore a bi-variable map of two

Abbreviations: CS, cyclostationary; SC, spectral correlation; SES, squared envelope spectrum; ACP, averaged cyclic periodogram; CMS, cyclic modulation spectrum; AT-CS, angle/time cyclostationary; OFACP, order-frequency averaged cyclic periodogram; OFCMS, order-frequency cyclic modulation spectrum; OFSCoh, order-frequency spectral coherence; IES, improved envelope spectrum; OFACCoh, order-frequency averaged cyclic coherence; OFCMCoh, order-frequency cyclic modulation coherence; ATCF, angle/time covariance function; TF-IPS, time/frequency instantaneous power spectrum; AF-IPS, angle/frequency instantaneous power spectrum; OLA, overlap add; FBS, filterbank summation; DFT, discrete Fourier transform; FFT, Fast Fourier transform

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http://dx.doi.org/10.1016/j.ymssp.2016.10.024
Received 20 June 2016; Received in revised form 9 September 2016; Accepted 22 October 2016
Available online xxxx

Nomenclature

Operators

$\mathcal{F}\{*\}$	Fourier transform
$\mathbb{E}\{*\}$	Ensemble average
$\|\|g\|\|^2$	Energy of g
$\otimes$	Numerical circular convolution
$F_{LIP}(h)$	Time reversed function of h
$COT_{\Delta t\to\Delta\theta}\{*\}$	Computed order tracking transform
$DFT_{n\to k}^{L}\{*\}$	Discrete Fourier transform over the length L

Theoretical variables

t	Time variable
τ	Time-lag variable
θ	Angle variable
f	Spectral frequency variable
α	Cyclic order variable
ω	Angular speed
W	Time duration of the realization
Φ	Angular sector spanned during the realization

Theoretical quantities

$R_{2X}(\tau, t)$	Instantaneous autocorrelation function of X
$\mathfrak{R}_{2X}(\tau, \theta)$	Angle/time autocorrelation function of X
$\mathfrak{R}_{2X}^{\beta}(\tau)$	(Angle/time) cyclic correlation function of X associated with the order β
$S_{2X}(f;t)$	Time-frequency instantaneous power spectrum of X
$\mathcal{S}_{2X}(f;\theta)$	Angle-frequency instantaneous power spectrum of X
$\mathcal{S}_{2X}^{\beta}(f)$	Order-frequency cyclic power spectrum of X associated with the order β
$\mathcal{S}_{2X}(f;\alpha)$	Order-frequency spectral correlation of X
$\gamma_{2X}(f;\alpha)$	Order-frequency spectral coherence of X
$I_{2X}(\alpha)$	Improved envelope spectrum of X
$S_{YZ}(f)$	Cross-power spectral density of signals Y and Z

Estimation variables

t_n	Discrete time variable
n	Time index
L_t	Digital length of the time signal
Δ_t	Sampling period
Δ_f	DFT resolution of the full length signal
Δt	Time resolution of the spectrogram
Δf	Spectral frequency resolution
$\Delta\alpha$	Cyclic order resolution
t_s'	Discrete time variable of the spectrogram (decimated)
f_k	Discrete spectral frequency variable of the estimators
α_i	Discrete cyclic order variable of the estimators
S	Number of shifting operations
S_θ	Digital length of the angle-frequency spectrogram over the angle-variable
$\theta[n]$	Angular profile
$\omega[n]$	Angular speed profile
$\overline{\omega}$	Mean speed in the record
ω_{min}	Minimal speed in the record
ω_{eq}	Equivalent speed in the record
ρ	Mean to minimal speed in the record
ω_{max}	Maximal speed in the record
$h[n]$	Tapering data-window
N_h	Tapering data-window length
W_h	Main-lobe effective bandwidth in bins
R	Hop size
ξ_Q	Variance reduction factor

Estimators

$\hat{\mathcal{S}}_{2X,L_t}^{(ACP)}(f_k;\alpha_i)$	Order-frequency averaged cyclic periodogram of X over a L_t-long record
$\hat{\mathcal{S}}_{2X,L_t}^{(CMS)}(f_k;\alpha_i)$	Order-frequency cyclic modulation spectrum of X over a L_t-long record
$\hat{S}_{YZ,L_t}^{(Welch)}(f_k)$	Welch estimator of the cross-power spectrum of signals Y and Z over a L_t-long record
$\hat{S}_{2X,L_t}(t_s', f_k)$	Time-frequency spectrogram of X over a L_t-long record
$\hat{\mathcal{S}}_{2X,L_t}(\theta_m, f_k)$	Angle-frequency spectrogram of X over a L_t-long record
$\hat{\gamma}_{2X,L_t}^{(ACP)}(f_k;\alpha_i)$	Order-frequency averaged cyclic coherence of X over a L_t-long record
$\hat{\gamma}_{2X,L_t}^{(CMS)}(f_k;\alpha_i)$	Order-frequency cyclic modulation coherence of X over a L_t-long record
$\hat{I}_{2X,L_t}^{(ACP)}$	ACP-based estimator of the improved envelope spectrum of X over a L_t-long record
$\hat{I}_{2X,L_t}^{(CMS)}$	CMS-based estimator of the improved envelope spectrum of X over a L_t-long record

frequency variables with different physical meanings, namely the *spectral frequency* and the *cyclic frequency*. The former describes the properties of the stationary carrier on which the cyclic information is traveling, while the latter describes the periodic hidden modulations in the signal. When applied to CS signals, the SC embodies a symptomatic distribution of spectral lines parallel to the spectral frequency axis and located at the modulation cyclic frequencies. The intensity of these lines varies continuously along the spectral frequency axis according to the power spectral density of the carrier. Interestingly, the relationship between the SC and the squared envelope spectrum (SES) has been established in Ref. [18] wherein the SES was proven to equal the integration/summation of the SC over the spectral frequency axis. Viewed differently, when read as a function of the cyclic frequency, the SC of a signal at a given spectral frequency equals the SES applied to the same signal narrowly band-pass filtered at the same frequency. Since then, the SES has been considered as a CS tool [14,24].

The estimation of the SC follows the classical lines of stationary spectral analysis and, particularly, those concerned with the estimation of the power spectrum. A primitive and simple estimator is the (cyclic) periodogram, known to be asymptotically inconsistent and, thus, disregarded in the literature. Similarly to the stationary case, the consistency of the estimator is practically ensured by two methods. The first one inserts a smoothing window in the calculation of the (cyclic) periodogram itself— e.g.