Mechanical Systems and Signal Processing 87 (2017) 340–364

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Review

Volterra-series-based nonlinear system modeling and its engineering applications: A state-of-the-art review

C.M. Cheng, Z.K. Peng*, W.M. Zhang, G. Meng

State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai 200240, PR China

ARTICLE INFO

Keywords:
Volterra series
General frequency response function
Nonlinear output frequency response function
Nonlinear model
Kernel identification
Convergence

ABSTRACT

Nonlinear problems have drawn great interest and extensive attention from engineers, physicists and mathematicians and many other scientists because most real systems are inherently nonlinear in nature. To model and analyze nonlinear systems, many mathematical theories and methods have been developed, including Volterra series. In this paper, the basic definition of the Volterra series is recapitulated, together with some frequency domain concepts which are derived from the Volterra series, including the general frequency response function (GFRF), the nonlinear output frequency response function (NOFRF), output frequency response function (OFRF) and associated frequency response function (AFRF). The relationship between the Volterra series and other nonlinear system models and nonlinear problem solving methods are discussed, including the Taylor series, Wiener series, NARMAX model, Hammerstein model, Wiener model, Wiener-Hammerstein model, harmonic balance method, perturbation method and Adomian decomposition. The challenging problems and their state of arts in the series convergence study and the kernel identification study are comprehensively introduced. In addition, a detailed review is then given on the applications of Volterra series in mechanical engineering, aeroelasticity problem, control engineering, electronic and electrical engineering.

1. Introduction

Nonlinear problems are very common, and have been researched by engineers, physicists, mathematicians and many other scientists. To model and analyze nonlinear systems and solve related problems, people have carried out extensive studies, and developed a variety of mathematical theories and methods, among which the Volterra series is one of the most widely used and well-established methods. It can be traced back to the work of the Italian mathematician Vito Volterra about theory of analytic functional in 1887 [1]. Then, Norbert Wiener applied his theory of Brownian motion to investigate the integration of Volterra analytic functional and firstly used it for system analysis in 1942 [2,3]. As a general method for the design and analysis of nonlinear systems, it came into use after about 1957. Using Volterra series, many nonlinear phenomena could be explained, but it was very complicated, and could only be applied to the analysis of some relatively simple nonlinear systems. This problem restricted its application in practical engineering and the progress of the application research was very slow. This situation continued until the 1990s, then, because of the development and popularization of computer technology, the application of Volterra series has been widely ranged from aeroelastic systems, biomedical engineering, fluid dynamics, electrical engineering, to mechanical engineering, etc. Especially during the last ten years, the global scholars have published nearly one thousand SCI papers about the theory and application of Volterra series, and the number of citations is over ten thousand, which shows the powerful vigor and broad application prospects in

* Corresponding author.
E-mail address: z.peng@sjtu.edu.cn (Z.K. Peng).

http://dx.doi.org/10.1016/j.ymssp.2016.10.029
Received 15 March 2016; Received in revised form 13 October 2016; Accepted 29 October 2016
Available online 05 November 2016

Nomenclature

FRF	frequency response function
GFRF	general frequency response function
NOFRF	nonlinear output frequency response function
OFRF	output frequency response function
AFRF	associated frequency response function
ALE	associated linear equation
NARMAX	nonlinear autoregressive moving average model with exogenous inputs
HBM	harmonic balance method
$h_n(\tau_1,...,\tau_n)$	the nth Volterra kernel function
$H_n(\omega_1, \ldots, \omega_n)$	the nth orderGFRF
$G_n(\omega)$	the nth order NOFRF
$U_n(\omega)$	the Fourier transform of the system input $u(t)$ raised to nth power
$G_n[x(t)]$	the nth Wiener G functional
$\lambda_1^{j_1} \ldots \lambda_{s_n}^{js_n}$	monomials in OFRF
$\Phi_{\lambda_1 \ldots \lambda_{s_n}}^{(n:j_1 \ldots j_{s_n})}(\omega_1, \ldots, \omega_n)$	coefficients of monomials $\lambda_1^{j_1} \ldots \lambda_{s_n}^{js_n}$ in OFRF
K_n	the nth nonlinear gain constant in AFRF

this research field. Unfortunately, although the number of research papers is very large, hitherto the related research books are only two or three copies [4–6], and there is still no one review paper which summarizes the related research achievements and status of Volterra series. These problems restrict the further promotion of Volterra series. In order to let the beginners master Volterra series faster and better, this paper tries to concisely and comprehensively introduce it, summarize the related research achievements, and discuss its application prospects.

2. Volterra series

Volterra series is one of the earliest approaches to achieve a systematic characterization of a nonlinear system. It is a powerful mathematical tool for nonlinear system analysis. Essentially, it is an extension of the standard convolution description of linear systems to nonlinear systems. Therefore, in order to help people understand the theory better, the convolution integral and its related concepts in linear systems are taken as references.

2.1. The definition in time domain

If a system is linear and time-invariant, then the linear input-output relation of the system can be represented by the convolution integral, which is shown as follows,

$$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)u(\tau)d\tau \tag{1}$$

where, $u(t)$ is the input, $y(t)$ is the output, Eq. (1) can be interpreted as Duhamel integral, and the system is determined uniquely by the impulse response function $h(t)$.

Implementing the Fourier transform at both the left and right sides of Eq. (1), the linear frequency domain relational expression between the system input and output can be obtained,

$$Y(\omega) = H(\omega)U(\omega) \tag{2}$$

where $U(\omega)$, $Y(\omega)$, $H(\omega)$ are the Fourier transform of $u(t)$, $y(t)$, $h(t)$, respectively, $H(\omega)$ is also known as the frequency response function (FRF). For a linear system, $H(\omega)$ or $h(t)$ includes all the information in the system.

In contrast, for nonlinear continuous time-invariant systems with fading memory, under zero initial conditions, if the energy of input signal $u(t)$ is limited, the system response can be represented by Volterra series [5,7–9]. It is an extension of Eq. (1) for linear systems to nonlinear systems, which can be represented as,

$$\begin{cases} y(t) = y_0 + \sum_{n=1}^{\infty} y_n(t) \\ y_n(t) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} h_n(\tau_1, \ldots, \tau_n) \prod_{i=1}^{n} u(t-\tau_i) d\tau_1 \cdots d\tau_n \end{cases} \tag{3}$$

where, $h_1(\tau)$, $h_2(\tau_1,\tau_2)$, ..., $h_n(\tau_1,...,\tau_n)$ are each order Volterra kernel functions, which are the extensions of the impulse response function for the linear system to the nonlinear system. In addition, generally, the equilibrium position of the system is set to be zero. This means that the DC term y_0 equals zero [10] Eq. (3) reveals that, if all the Volterra kernel functions except the first order are zero, the system degenerates into a linear system.

For the discrete nonlinear time invariant system, using Volterra series, it can be represented as [11–14],

$$y(k) = y_0 + \sum_{n=1}^{\infty} \sum_{m_1=1}^{\infty} \cdots \sum_{m_n=1}^{\infty} h_n(m_1, \cdots m_n) u(k-m_1) \cdots u(k-m_n) \tag{4}$$

where, $u(k)$, $y(k) \in R$, are the system input and output, respectively, $h_n(m_1,...,m_n)$ is the nth discrete Volterra kernel function.

A significant characteristic of the Volterra kernel function is the symmetry, which can be represented as,

$$h_n(\tau_1, \tau_2 \ldots, \tau_n) = h_n(\tau_2, \tau_1 \ldots, \tau_n) = \cdots = h_n(\tau_{i_1}, \tau_{i_2} \ldots, \tau_{i_n}) \qquad i_j \neq i_k$$
$$i_1, i_2, \ldots, i_n \in (1, 2, \ldots, n), j, k \in (1, 2, \ldots, n) \tag{5}$$